# The Modeling and Forecasting of Extreme Events in Electricity Spot Markets

**Abstract**

Primary concerns for traders since the deregulation of electricity markets include both the selection of optimal trading limits and risk quantification. These concerns came about as a consequence of unique stylized attributes exhibited by electricity spot prices, such as clustering of extremes, heavy-tails and common spikes. The authors of this paper propose self-exciting marked point process (SEMPP) models, which can be defined in terms of durations or intensities and which can capture these stylized facts. This approach consists of modeling the times between extreme events and the size of exceedances which surpass a high threshold. Empirical results for four major electricity spot markets in Australia show evidence of dependence between occurrence times of extreme returns. This finding is directly related to the future behavior of the stochastic intensity process for price spikes. In addition, the proposed approach provides a more accurate one-day-ahead value at risk (VaR) forecasting in electricity markets than standard stochastic volatility models.

**Keywords:** Extreme value theory, autoregressive conditional duration, ACD-POT, Hawkes-POT, forecasting risk measures

## 1. Introduction

Electricity spot markets are a challenging research area due to the unique characteristics of energy markets, such as the fact that any rise in demand or any drop in production needs to be addressed by the trader, often with a high marginal cost. This highly inelastic demand is often responsible for price jumps to extreme levels, a characteristic known as a “spike” in the literature. For instance, spot price spikes in Australia’s National Electricity Market (NEM) can peak at levels above AUD$1000/MWh, while it is also common to observe daily prices reaching levels between AUD$100/MWh and AUD$1000/MWh, each of which can be described as extremely high prices (Becker & Hurn, 2007; Christensen et al., 2012; Clements et al., 2013). Compared to other financial markets, risk management in the electricity market is a relatively new area introduced after the restructuring of the Australian electricity industry in the late 1990s. Most of the approaches to the electricity spot prices focus on predicting the series’ next prices (Conejo et al., 2005; Soares & Medeiros, 2008; Karakatsani & Bunn 2008; Chan et al, 2008), mainly because prices in the spot market are highly volatile and the spot price can spike to several hundred times the average price within a short period.

An assumption made in most of these approaches is the memory-free nature of the spikes’ behavior. However, current research on electricity prices have found evidence that the time between spikes has a significant impact on the likelihood of future occurrences (Christensen et al., 2009; Christensen et al., 2012; Clements et al. 2013). For this reason, the aim of this paper is to determine if the times between past extreme events of electricity spot price returns involve some information which can be used to forecast the future behavior of these returns.

The contribution of this paper to the existing state of knowledge in the modeling of extreme movements in electricity prices is twofold. First, we introduce a framework that captures the short-term behavior of extreme events in electricity spot price returns based on the most recent extreme event times and some exogenous covariates, which are believed to influence the behavior of these events. In particular, we include three components as covariates. The first covariate is the electricity load, which is characterized by a varying behavior due to customer portfolios as a result of the liberalization of the Australian electricity market. The second covariate is temperature, which can affect electricity demand and production. For example, in a dry year, if the main source is hydropower, it will not be possible to produce the normal amount of electricity. It would therefore be necessary to look for alternatives which may result in more expensive production methods (Golombek et al., 2012). The third covariate is the exceedance sizes of these spike prices returns, which, according to other stock market applications, could have a direct relation with the time of occurrence between these events.[1]

We propose a self-exciting marked point process (SEMPP) framework, which can be defined in terms of duration-based or intensity-based approaches. The duration-based approach corresponds to an autoregressive conditional duration peaks over threshold (ACD-POT) model (Herrera & Schipp, 2013), while the intensity-based approach corresponds to a Hawkes model for the exceedance times combined with a peaks over threshold model (Hawkes-POT) for the marks (Chavez-Demoulin et al., 2005; Chavez-Demoulin & McGill, 2012). These models allow us to concentrate only on these extreme events, rather than the entire dataset, providing risk measures via which traders can select optimal trading limits.

The authors of this paper also apply these new models to four major electricity markets in Australia: New South Wales, Queensland, South Australia and Victoria. The models are compared using a competing approach, the AR-APARCH-EVT, giving better results for the one-day-ahead value at risk (VaR) forecast out-sample.

The results obtained by this investigation indicate a significant improvement in the forecasting of different risk measures, overall accuracy and backtesting analysis. The most interesting result in this paper is the significant evidence of dependence between inter-exceedance times (the time between extreme events) in the markets, which are directly related to the intensity of the stochastic process of price spikes.

The rest of this paper is organized as follows: Section 2 presents a literature review dealing with electricity spot prices. Section 3 describes the proposed approach, introducing the ACD-POT

[1] Chavez-Demoulin & McGill (2012) showed that for high frequency data of stock markets, the impact of large losses on the intensity of the process is driven by the amplitude of the losses, and decreases in relation to the time between extreme events.

and Hawkes-POT models and their parameterizations. In Section 4, the Australian electricity markets are analyzed using the proposed methodology. Section 5 presents the conclusions.

## 2. Literature Review

The daily spot prices in the electricity markets exhibit interesting stylized facts at both intra-day and daily levels. These stylized facts include mean reverting, high volatility, price spikes and jump clustering (Karakatsani & Bunn, 2008; Klüppelberg et al, 2010). Price spikes present a common problem in electricity markets and, therefore, various approaches have been proposed in the literature to explain these. Three main frameworks are highlighted below.

The first considers modeling the spot price, or return trajectory, by means of classical time series. For instance, Conejo et al. (2005) present a review on autoregressive moving average (ARMA) and autoregressive moving average with exogenous inputs (ARMAX) models, while Higgs (2009) considers a generalized autoregressive conditional heteroskedasticity (GARCH) process. Other approaches address modeling the price spikes through Markov switching models (Kosater & Mosler, 2006; Becker & Hurn, 2007) or Markov diffusion processes (Higgs & Worthington, 2008; Meyer-Brandis & Tankov, 2008).

The second set of approaches focus on the estimation and forecasting of risk measures for both short- and long-trading positions in electricity markets. In particular, Vehviläinen and Keppo (2004), based on both stochastic consumption and prices, propose hedging strategies in the Nordic electricity market using VaR as a risk measure. Deng & Jiang (2005) introduce a Levy process-driven Ornstein-Uhlenbeck stochastic model for electricity prices, and propose VaR and expected shortfall (ES) as important measures of portfolio diversification. Byström (2005) and Chan & Gray (2006) compute VaR by means of extreme value theory (EVT) and adjust the volatility in the time series with a GARCH model.

The third alternative is focused on forecasting the likelihood of extreme price events, rather than on the trajectory of the price. In particular, Christensen et al. (2009) introduce a Poisson autoregressive framework and Christensen et al. (2012) present an autoregressive conditional hazard (ACH) model to forecast extreme price events. Similarly, Clements et al. (2013) proposed a semi-parametric nonlinear variant of the ACH model for forecasting spikes. The forecast results obtained using these approaches, based in discrete point processes, seem to be more accurate than the previous approaches. In particular, this semi-parametric nonlinear variant produces better results out-sample, which is attributable to the ability of the approach to capture the inherent nonlinearity in the spike process.

In the same line, the basic framework in our research is based on continuous SEMPP. In particular, we introduce some variants of the ACD-POT (Herrera & Schipp, 2013) and Hawkes-POT (Chavez-Demoulin & McGill, 2012) approaches, which model the dependence between the inter-exceedance times. Employing this method, we concentrate on forecasting the VaR rather than predicting spot prices. Through this model, a trader is able to plan a strategy for their bidding on the spot market for upcoming periods.

## 3. Methodology

### 3.1 A point process approach to extreme value theory

Let us assume we have observed the extreme returns $Y_1, \ldots, Y_n$ of a known electricity spot market. These extreme returns could be the consequence of price spikes observed, e.g., during periods of high inelastic demand, supply shifts or extreme temperatures. Moreover, assume that the observations are, for the moment, independent and identically distributed (iid) random variables with distribution function *F* and $M_n = \max\{Y_1, \ldots, Y_n\}$. We consider a point process approach to those observations with magnitudes exceeding a high, previously defined threshold $u > 0$.

A point process is a collection of random variables in some space; in our case, $\{(t_i, y_i): i = 1, \ldots, T\}$ defines a subset of observations where $t_i$ corresponds to the occurrence time of the extreme return $y_i$, whose magnitude is higher than the threshold $u$. We denote by $\Omega = (0,1) \times (u, \infty]$ the space for which this two-dimensional point process is defined.[2]

According to the classical point process approach to EVT (see Pickands, 1971; Smith, 1990), this two-dimensional point process approach corresponds to a non-homogeneous Poisson process with the intensity at the position $(t, y)$ given by:

[2] Notice that the time has been rescaled to the interval (0,1) to simplify the exposition.

$$\lambda(t,y) = \frac{1}{\sigma}\left(1 + \xi\frac{y-\mu}{\sigma}\right)_+^{-\frac{1}{\xi}-1}, \tag{1}$$

where $(x)_+ = \max\{0, x\}$ denotes the positive part of $x$. In this equation, $\sigma > 0$ is a scale parameter, while $\mu$ and $\xi \in \mathbb{R}$ are the location and shape parameters, respectively. Consequently, the corresponding intensity measure of this process for a subset $(t_1, t_2) \times (y, \infty) \subseteq \Omega$ is given by:

$$\Lambda\{(t_1,t_2) \times (y,\infty)\} = \int_{t_1}^{t_2}\int_{y}^{\infty} \lambda(r,l)\, dr\, dl = -(t_1 - t_2)\ln H(y)_{\xi,\mu,\sigma},$$

when $0 \leq t_1 \leq t_2 \leq 1$, and $H_{\xi,\mu,\sigma}$ is actually the generalized extreme value distribution given by:

$$H(y)_{\xi,\mu,\sigma} = \begin{cases} \exp\left\{-\left(1 + \xi\frac{y-\mu}{\sigma}\right)^{\frac{-1}{\xi}}\right\}, \xi \neq 0 \\ \exp\left\{-\exp\left(-\frac{y-\mu}{\sigma}\right)\right\}, \xi = 0. \end{cases}$$

Thus, this type of point process is completely characterized by its intensity. However, stylized facts, such as extreme price volatility, spikes, jump clustering, fat tails, autocorrelation and seasonality, exhibited by electricity spot markets make the direct use of this theory impossible. For this reason, we propose a more flexible framework that takes these stylized facts into account.

### 3.2 SEMPP approach to EVT

A SEMPP is a rich class of stochastic processes that allows us to use the information provided by the dynamic interactions among the inter-exceedance times of extreme events and their magnitudes. A SEMPP $N(t)$ is a point process in which some additional features are measured at each point $t$, and whose conditional intensity depends on the history of the process $\mathscr{H}_t = \left(\{t_1, y_1\}, \ldots, \{t_{N(t)-1}, y_{N(t)-1}\}\right)$. The key features of a SEMPP are: a ground point process $N_g(t)$ which only characterizes the stochastic process of the occurrence time of the extreme events; and the process of the marks, which describes the magnitude of the extreme events associated with the ground process. The conditional intensity of this class of process is given by:

$$\lambda(t, y|\mathscr{H}_t) = \lambda_g(t|\mathscr{H}_t)g(y|\mathscr{H}_t, t). \tag{2}$$

In our case, the ground process corresponds to the occurrence time of the extreme losses and the marks correspond to the size of an exceedance. Under this framework, the intensity (1) can be rewritten as:

$$\lambda(t,y) = \frac{\lambda_g}{\beta}\left(1+\xi\frac{y-u}{\beta}\right)_+^{-\frac{1}{\xi}-1}, \tag{3}$$

where $\beta = \sigma + \xi(u-\mu)$ is a redefined scale parameter. Observe that under this reparameterization, $\lambda_g$ corresponds to the rate of a Poisson process of the exceedances of the threshold $u$ via:

$$\lambda_g(t|\mathscr{H}_t) = \Lambda\{(0,1)\times(y,\infty)\} = -\ln H(y)_{\xi,\mu,\sigma} \coloneqq \lambda_g,$$

while the rest of (3) defines the probability density function of the marks:

$$g(y)_{\xi,\mu,\beta} = \begin{cases} \frac{1}{\beta}\left(1+\xi\frac{y-u}{\beta}\right)_+^{-\frac{1}{\xi}-1}, & \xi \neq 0 \\ \frac{1}{\beta}\exp\left(-\frac{y-u}{\beta}\right), & \xi = 0, \end{cases} \tag{4}$$

which by definition is approximately a generalized Pareto density (GPD) function.[3]

Since inter-exceedance times of extreme events in electricity spot markets are irregularly spaced and show clustering, we propose that a SEMPP can be adopted for the purpose of spike modeling.[4] In particular, we consider two alternatives for the specification of a SEMPP: the duration-based (ACD-POT model) and the intensity-based approach (Hawkes-POT).

### 3.2.1 ACD-POT model

In the ACD-POT approach, the conditional intensity function of the ground process is modeled using the duration of the inter-exceedance times $x_i = t_i - t_{i-1}$. The idea is to standardize the durations utilizing the most recent history, with $\psi_i$ serving as conditional mean function of the current duration $E(x_i|\mathscr{H}_t) \coloneqq \psi_i$ and $\varepsilon_i = \frac{x_i}{\varphi(\psi_i)}$ serving as the standardized durations. The complete ground process for the ACD-POT model is defined by:

$$\lambda_g(t|\mathscr{H}_t) = \lambda_0\left(\frac{t-t_{N(T)}}{\varphi(\psi_{N(T)})}\right)\frac{1}{\varphi(\psi_{N(T)})}, \tag{5}$$

[3] See Theorem 3.4.13 Embrechts et al. (1997).
[4] See for instance Hautsch (2012) for a review of these models.

where $\varphi$ is a positive function that standardizes the durations, and $\lambda_0$ is a baseline function which corresponds to a hazard function, depending on the selected probability distribution function for the standardized durations and the specification of $\psi_i$.

Consider, first, the specification of the conditional mean function. The main types of ACD parameterizations that have been suggested in the financial econometrics literature are the linear ACD model (Engle & Russell, 1998) and the Logarithmic ACD *(*LogACD*)* model (Bauwens & Giot, 2000)[5]. In our setup, the conditional mean function takes the following forms:

$$\psi_i - \vartheta_i = w + \sum_{j=1}^{p} a_j x_{i-j} + \sum_{j=1}^{q} b_j (\psi_i - \vartheta_i) \tag{6}$$

for the linear ACD model, and

$$\psi_i - \vartheta_i = w + \sum_{j=1}^{p} a_j \ln x_{i-j} + \sum_{j=1}^{q} b_j \big(\psi_{i-j} - \vartheta_{i-j}\big) \tag{7}$$

for the LogACD.[6] In this type of model, $p$ and $q$ are non-negative integers indicating the order of the autoregressive terms, while $a_j$, $b_j$ and $w$ are constant coefficients. Observe that these models also include a set of covariates represented by the component $\vartheta_i$, which will be described in detail in Section 4.2.[7]

We also need to specify the probability distribution function of the standardized durations. Several alternative choices are plausible, with high degrees of flexibility at the expense of high complexity. In this paper, the authors consider two alternatives; the generalized gamma (Lunde, 1999) and the Burr (Grammig & Maurer, 2000). The generalized gamma distribution has the probability density function:

$$f(x|\lambda, k, \gamma) = \frac{\gamma x^{k\gamma - 1}}{\lambda^{k\gamma}\Gamma(k)} \exp\left\{-\left(\frac{x}{\lambda}\right)^{\gamma}\right\}, \text{ where } \lambda > 0, \gamma > 0 \text{ and } k > 0.$$

To obtain standardized durations with mean one, we have to fix $\lambda = 1$, which implies that $\varphi(\psi_i) = \psi_i \Gamma(k)/\Gamma(k + 1/\gamma)$. The second option is the Burr, whose probability density function is:

[5] Both Linear ACD and LogACD present only linear relations between observations. Other alternatives, such as Box-Cox ACD and Exponential ACD, show non-linear relations, but were ultimately disregarded in the analysis as they failed to produce more significant results over linear models.

[6] Observe that the LogACD model suggests a multiplicative relationship on durations.

[7] For proof that both models are stationary and that the unconditional expected duration exists for each model, we refer to Hautsch (2012).

$$f(x|\lambda, k, \gamma) = \frac{\lambda k t^{k-1}}{(1+\gamma^2 \lambda t^k)^{\gamma^{-2}+1}}, \qquad \text{where } \lambda > 0, \gamma > 0 \text{ and } k > 0.$$

In this case, $\varphi(\psi_i) = \psi_i \frac{\gamma^2(1+1/k)\Gamma(\gamma^{-2}+1)}{\Gamma(1+1/k)\Gamma(\gamma^{-2}-1/k)}$, where $0 < \gamma^{-2} < k$. Both distribution functions have shown a high flexibility in the context of irregularly spaced modeling due to the non-monotone behavior of their hazard functions. This feature is of particular importance if we are interested in models, which after one observed electricity price spike, can adapt rapidly in periods where temporarily high demand must be met by generating its power at premium prices.

At this point, after the specification for the ground process, we are ready to parameterize the density of exceedance sizes of these spike price returns. Recent studies in SEMPP applied in the context of financial risk have shown that models, where the scale parameter β of the GPD in (4) varies over time, display a better fit and performance in backtesting (Chavez-Demoulin et al., 2005; Chavez-Demoulin & McGill, 2012; Herrera & Schipp, 2013). We propose a linear specification for the scale parameter:

$$\beta(t|\mathcal{H}_t) = \beta_0 + \beta_1 \lambda_g(t|\mathcal{H}_t), \tag{8}$$

where the exceedances are conditionally generalized Pareto distributed, given the exceedance history up to the time of the mark, with the coefficients being positive.[8] Replacing (5) and (4) in (2), we finally obtain the conditional intensity function defined for an ACD-POT model:

$$\lambda(t, y|\mathcal{H}_t; \theta) = \lambda_0 \left(\frac{t - t_{N(T)}}{\varphi(\psi_{N(T)})}\right) \frac{1}{\varphi(\psi_{N(T)})\beta(t|\mathcal{H}_t)} \left(1 + \xi \frac{y-u}{\beta(t|\mathcal{H}_t)}\right)_+^{-1/\xi - 1}. \tag{9}$$

### 3.2.2 Hawkes-POT Model

Introduced preliminarily in Chavez-Demoulin et al. (2005) and detailed recently in Chavez-Demoulin & McGill (2012), the Hawkes-POT model is an alternative to the ACD-POT model. The main advantage of the Hawkes-POT model as opposed to the ACD-POT approach is that we directly parameterize $\lambda_g(t|\mathcal{H}_t)$ which allows for updating of the intensity process whenever required. We propose the direct inclusion of the covariates in the self-exciting function of the

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[8] Other specifications for scale parameter have also been investigated, but this simple approach seems to be the most robust and with an easy economic interpretation (for other specifications, see Herrera & Schipp, 2013).

Hawkes process, which is governed by the sum of exponential functions of the covariates and the time to all previous extreme events:

$$\lambda_g(t|\mathcal{H}_t) = w + a \sum_{j=1}^{N(t)} \exp\{\vartheta_i - b(t - t_j)\}, \tag{10}$$

where all parameters have positive values. The parameter $w$ corresponds to the baseline, the parameter $b$ determines the decay function of influence of past extreme events and $a$ determines the amplitude. Also here, the component $\vartheta_i$ represents a set of covariates.

Observe that, unlike the ACD-POT approach, where the covariates influence the conditional mean function, the Hawkes-POT model considers that the covariates affect the intensity of the ground process directly. Similar to the scale parameterization in (8) for the ACD-POT approach, Chavez-Demoulin et al. (2005) propose the following structure for the Hawkes-POT model:

$$\beta(t|\mathcal{H}_t) = \beta_0 + \beta_1 \sum_{j=1}^{N(t)} \exp\{\vartheta_i - b(t - t_j)\}. \tag{11}$$

Both specifications for the scale parameter (8) and (11) are justified under the assumption that both the frequency and the magnitude of extreme events increase during periods of turmoil in electricity spot markets simultaneously. Finally, the estimation of the ACD-POT and the Hawkes-POT models is performed by maximizing the log-likelihood in terms of the conditional intensity, as follows:

$$l = \sum_{i=1}^{N(T)} \ln \lambda_g(t_i|\mathscr{H}_t) - \int_0^T \lambda_g(s|\mathscr{H}_s)ds + \sum_{i=1}^{N(T)} \ln g(y|\mathscr{H}_t, t).$$

### 3.2.3 Conditional risk measures based on SEMPP

Chavez-Demoulin & McGill (2012) and Herrera & Schipp (2013) show how electricity companies can track their exposure to individual market risk factors by using measures such as VaR estimated by means of the SEMPP approach. The conditional VaR for a confidence level $\alpha$ is obtained as follows:

$$VaR_\alpha^{t+1} = u + \frac{\beta(t|\mathscr{H}_t)}{\xi}\left(\left(\frac{1-\alpha}{\lambda_g(t|\mathcal{H}_t)}\right)^{-\xi} - 1\right).$$

The main advantage of this approach is its simplicity, which expresses the total risk exposure in a single number.

## 4. Application to Australian electricity spot markets

### 4.1 Data description

In this paper, the authors concentrate on four major electricity markets in Australia: New South Wales (NSW), Queensland (QLD), South Australia (SA) and Victoria (VIC). The data for the estimations correspond to the daily Regional Reference Price (RPP, in AUD$/MWh), the maxima temperatures in degrees Celsius, and the load, which is defined as the electrical power requirement (MW).[9]

The sample covers 3228 daily observations, dating from March 1, 2001 to December 31, 2009; a second sample with 1096 observations, dating from January 1, 2010 to December 31, 2012, is used for backtesting. Table 1 presents descriptive statistics for these log-prices. We report distributional statistics of log-prices instead of the spot price itself, because the models will be based on daily log-price returns. These series of returns exhibit the following stylized facts: extreme values for maxima and minima, and skewness and heavy-tails, denoted by excess kurtosis. Moreover, we test if the log-prices are normally distributed and uncorrelated through the Jarque-Bera and the Box-Pierce statistics, respectively. All the log-prices analyzed reject the null hypothesis. In relation to the analyses of stationary by means of the ADF-test, all log-prices reject the null hypothesis of non-stationary at the 0.01 level of significance. These results coincide with the results of other authors (Higgs & Worthington, 2008; Higgs, 2009). For a compressed and well-documented study of the stylized facts of spot prices in the Australian wholesale electricity market we refer to Becker & Hurn (2007).

< Insert Table 1 about here>

### 4.2 Covariates driving price spikes

An important attribute of electricity spot prices since the deregulation is that prices are now determined according to the fundamentals of supply and demand. In short, while electricity is

[9] The data were obtained from the website of the Australian Energy Market Operator (AEMO) and from the Australian Bureau of Meteorology on a daily basis.

continuously generated with some infrastructural limitations (e.g., constraints of storage and transmission lines) and influenced by exogenous factors (e.g., weather conditions and seasonality), the sale prices to consumers are fixed. Therefore, it is clear that the extreme behavior of electricity price spikes is in part determined by these other factors.

According to various authors, these exogenous factors are important for explaining the intensity of spikes in spot markets (Christensen et al., 2012; Golombek et al., 2012; Clements et al., 2013). In this investigation, we incorporate three covariates. First, the exceedance sizes (or marks) of these log-prices returns $(y_i - u)$, which, according to the results obtained in other financial markets (Chavez-Demoulin & McGill, 2012; Herrera & Schipp, 2013), have a direct impact on the intensity or frequency of these extreme events. Second, the logarithms of electricity load (*L*), which represents the contemporaneous demand. Third, the daily maximum temperature observed (*T*), as a proxy for weather conditions. The last two covariates are constructed by detrending them using a simple linear trend model, while preserving obvious seasonal fluctuations and abnormal load and temperature events.[10]

In contrast to Christensen et al. (2012), we include them in the conditional mean function of the ACD-POT model and in the kernel function of the Hawkes-POT model. In this way, the covariates will influence both models in terms of an infinite lag structure by means of the following function:

$$\vartheta_i = c_1 L_i + c_2 T_i + c_3 (y_i - u),$$

where the parameters $c_1, c_2$ and $c_3$ are unrestricted.[11]

### 4.3 Seasonality

As pointed out by different authors, the first step in defining a model for electricity spot price dynamics consists of describing the seasonal component appropriately. This can be done in a number of ways, including principally sinusoidal functions (Lucia & Schwartz, 2002; Weron et al.,

[10] For the period analyzed, we notice a clear correlation between daily maximum and minimum temperatures, causing non-invertibility of the Hessian matrix of the models. For this reason, we only consider the covariate with the highest level of explanatory power; in this case, the daily maximum temperature.

[11] Our results suggest that this autoregressive specification captures the effect of these covariates in a better way, according to goodness-of-fit tests.

2004; Pilipovic, 2007) and dummy variables (Lucia & Schwartz, 2002; Higgs, 2009). In this paper, the authors follow a combination of both, similar to the proposal by De Jong (2006). We assume that the log-spot price $P_i$ is composed of two independent parts: a seasonal component $S_i$ and a stochastic component $Z_i$, i.e., $P_i = S_i + Z_i$. For the seasonal component we propose the following model:

$$S_i = \sum_{j=1}^{7} \phi_j D_{i,j} + \phi_8 \sin\left(\phi_9 \frac{2\pi i}{365.25}\right) + \phi_{10} EWMA_{i-1}^{\eta},$$

where $D_{i,j}$ are dummies for individual week days. We also include a sinusoidal function to capture the seasonality over the year, and an exponentially weighted moving average (EWMA):

$$EWMA_i^{\eta} = (1-\eta) P_{i-1} + \eta EWMA_{i-1}^{\eta}$$

to control the trend showed by log-prices. After some experimentation, and to avoid overfitting, we choose a decay factor $\eta = 0.70$ for the log-spot prices analyzed. The deseasonalized log-spot prices $Z_i$ are obtained by subtracting the calculated seasonal component to the log-spot prices.[12] Finally, the deseasonalized log-spot price returns are obtained as $R_i = Z_i - Z_{i-1}$. Since we concentrate on the left tail for risk management, the negative log-deseasonalized log-spot price returns $R_i$ are used for the analysis.

### 4.4 Estimating the SEMPP models

#### 4.4.1 Stylized facts and threshold selection

One requirement for applying EVT to the log-spot price returns is the choice of a sufficiently high threshold, $u > 0$, without compromising the variance of the sample. We selected the Hill plot, a common estimator for finding an optimal threshold to help choose this threshold (e.g., Embrechts et al., 1997). These Hill estimates indicate that, for this application, a good compromise for the threshold $u$ is 10% of the sample. As reported by Christensen et al. (2012), this threshold corresponds in prices to AUD$100/MWh, which seems to be the most informative threshold with which to define spikes according to market participants.

[12] The results of the estimation of these seasonal components are available upon request.

Figure 1 displays some of the stylized facts which motivate this study. On the left, we observe the marks or excesses over the threshold $u$ previously defined for each return and the apparent clustering behavior in each return. This clustering hypothesis is validated by the autocorrelation exhibited by the inter-exceedance times. According to point process theory, if the extreme observations were independent events over time, the inter-exceedance times would be exponentially distributed, which is a fundamental assumption of a Poisson process model. However, the evidence in the QQ-plots displayed on the right of Figure 1 completely contradicts this idea. Indeed, the inter-exceedance times display a significant dependence among each other, which could be important in explaining the clustering behavior at extreme levels. These findings coincide with the results obtained in Christensen et al. (2012), where the intensity of the spikes over time exhibit a significant dependence on the history of the process.

< Insert Figure 1 about here>

**4.4.2 Duration-based or intensity-based driving process?**

Results of the estimation of the SEMPP models for all return series analyzed are presented in Table 2. Concerning the overall fit, the results lead markedly toward favoring the ACD–POT model approach, though between the two different conditional mean function specifications, we did not find significant differences.[13] Importantly, all the coefficients related to these conditional mean functions and probability distribution functions seem to be highly significant. However, the results suggest a marked preference for the standardized residuals to follow a Burr distribution over a generalized gamma distribution function with a non-monotone behavior ($k > 1$).

In relation to results obtained for the Hawkes-POT model, we observe that low values for standard deviations of parameter estimates also confirm the good fit of these. The rate of new exogenous extreme events in electricity markets is determined by the background intensity $w$, while the kernel at the right of (10) accounts for the mutual excitations of past extreme events.[14] Observe

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[13] In terms of fit, all models ACD(p,q)-POT prefer the orders of the persistence *p*= 1 and *q* =1, over more complex specifications. These results coincide with those obtained by Christensen et al. (2012) for an ACH model.

[14] A recent attempt to establish a link between the ACD and Hawkes models is the work of Filimov et. al (2013) by means of effective measure of endogeneity.

that for the markets analyzed, the range of exogenous extreme events corresponds to 50% to 70% of the sample, which means that on average, 40% of the extreme events are due to endogenous triggering effects rather than to genuine new price spikes.[15]

Results for the risk measures induced by ACD-POT and Hawkes-POT models share many similarities. For instance, in Figure 2, VaR estimations at confidence level 0.99 fluctuate in a similar range and show qualitatively similar clustering of events, even when the covariates affect these markets in different ways.

< Insert Figure 2 about here>

**4.4.3 The impact of explanatory covariates**

Table 2 also reveals other important results related to the inclusion of explanatory covariates. For New South Wales, the covariates added little to the conditional mean function behavior for the ACD-POT models, making the majority of coefficients not statistically significant. The same occurs when we include these covariates directly in the conditional intensity function of the Hawkes-POT model.

In the case of Queensland, the inclusion of explanatory variables seems to play an important role, with all of them being highly significant in the ACD-POT and Hawkes-POT approaches. We observe that the size of exceedance has a negative impact on the conditional mean function of the ACD-POT model, while the same occurs for the Hawkes-POT model but with the factor load.

For South Australia, the load factor and temperature display a positive impact on the intensity of the ground process for the Hawkes-POT model, while for the ACD-POT, there is no evidence of covariate influences on the conditional mean function.

Finally, for Victoria, only the load factor drives the intensity of the ground process by means of the inter-exceedance times in the ACD-POT models, while all explanatory variables are significant for the Hawkes-POT model.

[15] Indeed, for Hawkes-POT models the background intensity belongs $w \in [0.05, 0.07]$, while we consider the threshold of extreme events to be 10% of the sample.

These findings corroborate that the ground point process' intensity for returns in these markets is related to its past realizations and in some cases to other explanatory covariates in concordance with results of previous studies (Christensen, 2012; Clements et al., 2013; Mount et al., 2006).

### 4.4.4 Are the intensity and size of these events related?

Basically, the hypothesis behind the scale specification in (8) and (11) follows the logic that an augment in the frequency of extreme events entails an increase in the size of these exceedances during periods of turmoil. In accordance with it, the results of the GPD estimates justify the influence of the conditional intensity of the ground process over the scale behavior ($\beta_1 > 0$) for both SEMPP approaches. This finding has also been addressed in other financial markets (Herrera & Schipp, 2013; Chavez-Demoulin & McGill, 2012). Furthermore, for both models, we test the fit of the marginal distribution of marks. Smith (1990) proposes the following test:

$$W_i = \frac{1}{\xi}\ln\left(1 + \xi\frac{y_i - u}{\beta(t|\mathcal{H}_t)}\right).$$

If the assumed model is correct, then $\{W_i\}_{i\geq 1}$ are independent exponentially distributed random variables with mean one. We test if $\{W_i\}_{i\geq 1}$ are unit exponential variables and approximately independent by means of a Box-Ljung test ($BL_W$) and a Kolmogorov-Smirnov test ($KS_W$). According to the p-values for these two tests, displayed in Table 3, the dynamic specification for the scale parameter for both approaches seems to be appropriate for every return analyzed.

< Insert Table 3 about here >

Finally, Figure 2 shows in-sample estimates for VaR at a confidence level of $\alpha = 0.99$ for all models in every log-return series. From top to bottom, the figures correspond to NSW, QLD, SA and VIC markets, while from left to right we observe the proposed models, namely the linear ACD-POT and the LogACD-POT, both with generalized gamma and burr distribution functions of probability for the standardized residuals, and finally the Hawkes-POT model. The " $\times$ " symbol above the estimates for the VaR in Figure 2 indicates violations at this confidence level. Observe how we can better capture the magnitude of extreme events VaR for electricity markets when we take into account the inherent autoregressive or auto-excited behavior of inter-exceedance times and explanatory covariates in the intensity of the spiking process, providing practitioners an efficient way to calculate risk contributions in practice.

### 4.5 A simple benchmark model

We propose an AR(7)-APARCH(1,1)-EVT model that assumes innovations $z_t = \varepsilon_t/\sigma_t$ with skewed t-Student conditional distribution which has been demonstrated to be the best alternative (Chan & Gray, 2006)[16]. Under this framework, the AR(7) model captures the weekly seasonality for the mean

$$r_t = \mu + \sum_{j=1}^{7} \phi_j r_{t-j} + \varepsilon_{t,}$$

where $\mu$ and $\phi_j$ are parameters estimated, $\varepsilon_t$ are the residuals of the AR model and $r_t$ are log-price returns. As a second step, we propose an APARCH(1,1) model that captures the leverage effects observed in the literature (Knittel & Roberts, 2005; Chan & Gray, 2006):

$$\sigma_t^2 = \omega + \alpha_1(|\varepsilon_{t-1}| - \gamma_1\varepsilon_{t-1})^\delta + \beta_1\sigma_{t-1}^\delta,$$

where $\delta > 0$, $-1 < \gamma_i < 1$.

The estimation results are displayed in Table 4. Similar to Higgs and Worthington (2005), we find some variation between electricity markets in Australia in relation to the estimated power term of the APARCH model. With the exception of Victoria, all the other markets seem to be significantly different from a Taylor-Schwer model (TS-GARCH implies $\delta = 1$) or a simple GARCH model ($\delta = 2$). In the case of Victoria, a GARCH model should be enough. Furthermore, we observe that for all returns analyzed, there exists evidence of a positive leverage effect ($\alpha_1 > 0$), which means that past negative returns shock have a deeper impact on current conditional volatility than past positive shocks. Actually, this AR-APARCH-EVT model was the best model fitted to each return analyzed and it is in line with the framework proposed in other energy markets (Higgs and Worthington, 2005; Feng et al., 2012; Marimoutou et al., 2009). In relation to the shape and scale parameter estimates, by fitting a GPD to the standardized residuals, all of the estimates are

[16] The determination of the ARMA order and the APARCH order has been realized using a traditional model selection criterion, the Akaike Information Criterion (AIC).

statistically significant. For the estimation of VaR in the backtesting, this model is re-estimated on a daily basis.

< Insert Table 4 about here >

### 4.6 Value at Risk Forecasting

So far, the analysis has been focused on the results in-sample. However, a notably important measure of performance relies on testing the proposed model on certain out-of-sample data; this importance is due to the trading agents' natural use of risk measures. We selected the most recent three years of daily observations; that is, from January 1, 2010 to December 31, 2012. The backtesting is implemented for a one-day-ahead VaR forecast, which means that the parameters of the models are estimated using the in-sample period of observations (January 1, 2010 to December 31, 2012) more than most actual daily observations of log-returns. This also includes the observed explanatory covariates for the forecasting.[17]

< Insert Table 5 about here>

Before the estimation of VaR for the backtesting sample, we provide a short description of a set of statistic tests which are used to determine the accuracy of VaR forecasting.

#### 4.6.1 Forecast evaluation framework

To determine the accuracy of VaR estimates, we consider a set of statistic tests. A detailed explanation of each test can be found in Kuester et. al (2006). We define the failures as the number of days whose return exceed the estimated VaR:

$$I_t(\alpha) = \begin{cases} 1 & if\ R_t > VaR_\alpha^t \\ 0 & else \end{cases}$$

while $Hit_t(\alpha) = I_t(\alpha) - \alpha$ is the de-meanded process on $\alpha$ associated to $I_t(\alpha)$.

The first three tests were proposed by Christoferssen (1998): a test of unconditional coverage ($LR_{uc}$) to test whether the fraction of violations obtained for the VaR is indeed the expected fraction; a test of independence ($LR_{ind}$) to test independence of the failures; and a

[17] Alternatively, we could have used a forecasting approach for the load and the temperature as was done by Weron (2006) and Christensen et al. (2012). However, we wish to keep the simple structure of the actual model.

conditional coverage ($LR_{cc}$) test (which is a combination of the last two tests) to determine independence and correct coverage. Moreover, we implement two more dynamic tests based on linear regression models—the Dynamic Quantile tests proposed by Engle & Manganelli (2004). The first is the dynamic quantile $Hit_t(\alpha)$ ($DQ_{hit}$) test, where the regressors are the lagged indicator function of failures (the hits), while the second dynamic test, the dynamic quantile VaR ($DQ_{VaR}$) test uses, in addition, the contemporaneous VaR estimates. In the empirical application, we display only the p-values obtained. For every one of these statistical analyses, a p-value > 0.05 will be considered as an approved test.

#### 4.6.2 Results for the VaR forecast

Table 5 exhibits the p-values for these accuracy tests for the VaR at three different confidence levels (0.95, 0.99, 0.999). In general, the proposed models perform well under new observations, with most of the alternative models satisfying all tests for adequate VaR estimation in every case and with no significant difference in performance among them.

For NSW log-returns the best model approving all tests is an ACD-POT model with Burr distribution for the standardized inter-exceedance times, followed by the Hawkes-POT approach. Observe that, contrary to the Hawkes-POT model, most of the ACD-POT models provide a lower number of VaR failures than expected for the confidence level 0.95. These results can be explained by a number of extreme observations that have been underestimated in the backtesting sample. However, for ACD-POT models, the failures of the VaR at this confidence level do not display any temporal pattern, in contrast to the results showed by the Hawkes-POT model.

For the QLD log-returns, two ACD-POT models (linear ACD-POT and Log-ACD-POT) and the Hawkes-POT model exhibit a perfect score in the accuracy test. The bad results obtained for the other two models is due mainly to the misspecification of the distribution assumption for the standardized inter-exceedance times, which did not capture the clustering behavior of these extreme events, displaying evidence of non-independence for the VaR failures at confidence level 0.95.

Regarding the SA market, the results are very promising for all SEMPP models during the backtesting. All models tend to estimate the frequency of extreme returns correctly, producing independent VaR failures at all confidence levels.

Finally, the results in the backtesting for the VIC market exhibit the poorest performance, being that the linear ACD-POT is the best option besides the distributional assumption for the standardized inter-exceedance times. It passes 13 out of the 15 tests for both models, whereas the third best specification passes only 8 of the 15, with the major problem being the consistent behavior of the VaR failures to gather into clusters.

Table 5 also presents the results for the AR-APARCH-EVT benchmark model. For most of the markets, the results show that the benchmark approach is inadequate for estimating VaR in these markets, supported by the unconditional coverage tests ($LR_{uc}$) for low confidence levels. This result demonstrates the difficulty of investigating the stylized facts for electricity spot price returns using the classical time series framework, where we assume the same behavior in the observations during periods of low demand, as well as of unexpectedly high demand.

In light of these results, the ACD-POT approach provides a suitable alternative for short-term forecast of risk measures in electricity spot markets characterizing the behavior of extreme events. In particular, empirical results of ACD-POT models to the one-day-ahead forecast show a superior performance in comparison to volatility models whose behavior aims to capture the trajectory of price spikes more than the clustering behavior of extremes.

## 5. Conclusions

The aim of this investigation was to determine if the time between past extreme events in electricity spot price returns is a determinant of future price spike behaviors. To this end, we propose the SEMPP approach, which exhibits a high flexibility and focuses on capturing the relations between inter-exceedance times of extreme events rather than the whole time series, together with a set of covariates.

In an empirical application to the Australian electricity market, the SEMPP models were evaluated in terms of the overall test of all parameters, goodness-of-fit statistics and accuracy of prediction for risk measures. In particular, we compare the ACD-POT and Hawkes-POT approach for the estimation and forecasting of the VaR. Finally, we compare the SEMPP models with a common alternative; a combination of autoregressive and stochastic volatility models with refinements of EVT, an AR-APARCH-EVT model. For most of the returns analyzed, the SEMPP approach gives better results.

One of the more significant findings to emerge from this study is that there exists evidence of dependence between inter-exceedance times of extreme returns in these markets, which is directly related with the future behavior of the stochastic intensity process for price spikes. Furthermore, the inclusions of covariates, such as the influence of load or temperature factors, also have a positive impact on the intensity of the ground process for these extreme events. An interesting future research possibility is to directly model the price spikes using a regime-switching approach for the conditional intensity of the ground process in order to respond to these shifts more rapidly. Another possible extension is to analyze these markets as a group by means of a multivariate point process model, taking into account market specific information (market design, market participants, demand and weather-related covariates) that might increase the model's explanatory power.

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