Modeling Extreme Risks in Commodities and Commodity Currencies

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joint work with

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Motivation



- In many countries there are principal commodities that dominate a significant share of total exports.
- The prices at which these commodities are traded are crucial to the economic performance and generally have a significant effect on exchange rates behavior [Edwards, 1986].

Motivation

- Previous studies have analyzed the relationship between commodity prices and exchange rates, often using
 - Cointegration and error correction models [Hatzinikolaou and Polasek, 2005, Wu, 2013, Kohlscheen, 2010, Bjørnland and Hungnes, 2005], and
 - Tests for causality [Chen et al., 2010, Choudhri and Schembri, 2014, Chan et al., 2011].
- Overall, a relationship flowing from commodity to currency markets is often found to exist [Cashin et al., 2004, Chen and Rogoff, 2003, Bowman et al., 2005, Bodart et al., 2012, Kato et al., 2012].
- According to a 2010 study on commodity currency titled "Can Exchange Rates Forecast Commodity Prices?" by Chen et al. [2010], exchange rates of commodity currencies can predict future global commodity prices.

Motivation

Questions of interes from the point of view of joint behavior at extreme levels:

- The underlying relationship between extreme events and comovements during financial turmoil periods [Belhajjam et al., 2017].
- Examining the complex behavior of these markets at extreme levels is not a straightforward task, mainly because it involves capturing stylized facts [Hung et al., 2008, Liu and Tang, 2011, Delatte and Lopez, 2013, Aboura and Chevallier, 2015].
- A large literature presents tools to measure risk in periods of stability, but these are not valid in turbulent times [Massacci, 2016].
 ⇒ Potential misestimation of the tails of the probability distribution [Kellner and Gatzert, 2013].
- It is essential to employ methods that can adequately capture the complex relationship between these markets in extreme economic conditions.

Research Question

Do intensity based models (with causal links) provide superior forecasts of extreme risk relative to the volatility and correlation based approach (only contemporaneous links)? Or are these approaches complementary?

- In addressing this question it will become evident if the unidirectional link to currency markets is important when forecasting risk.
- This comparison will also shed light on the question of whether using the full set of returns (correlation) or focusing only on the extreme losses is most beneficial for forecasting extreme risk.
- The methodologies proposed here along with the empirical results will provide participants in the commodity and currency markets, commodity traders, producers and wholesale consumers, a deeper understanding of the extreme risks they face.

Data Description

- The financial series are daily return observations on
 - Australian Dollar (AUD) vs Gold
 - Canadian Dollar (CAD) vs Brent oil.
- All series were obtained from Bloomberg. As extreme losses are analyzed here, negative logarithmic returns of the financial price series $X_t = -\ln(P_t/P_{t-1})$ are utilized.
- In order to determine the predictive ability of the models, the database was divided into two periods.
 - The first 10 years are used for estimation and model fit, with observations ranging from 4 January 2005 to 31 December 2014.
 - For the backtesting sample, the period from 1 January 2015 until 31 December 2016 is used.

Intensity vs Volatility



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Intensity vs Volatility



Outline

Motivation

Methodology

Empirical Analysis

Conclusions

What are extreme events

Why we need to study extreme events?

Extreme events in finance do not consist in stock market crashes only. Extreme events occur as well in the normal run of financial institutions, or in other words, in their daily, trading and non-trading operations.

What distinguishes extreme events?

- Maxima/minima
- Magnitude
- Rarity
- Impact/losses



"Big movements in the commodity are associated with big movements in the terms of trade, which then should have currency consequences"

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Extreme Value Theory

Let $x_t \in \mathbb{R}$ are iid rv with finite mean μ , finite variance σ^2 and continuous distribution function F and upper support point of x_0 such that $\lim_{x\to x_0} F(x) = 1$. Let

$$S_T = x_1 + \cdots + x_T$$
.

Then,

$$\lim_{T\to\infty}P\left(\frac{S_T-T\mu}{\sigma\sqrt{T}}\leq x\right)\to\Phi(x)\,,$$

where Φ is the distribution function of the standard normal distribution.

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In Extreme Value Theory we wonder if there is any limit considering only the maximum, i.e,

 $M_T = \max\left(x_1, \ldots, x_T\right)$

Extreme Value Theory

Choose parameters $a_T > 0$ and $b_T \in \mathbb{R}$ such that when $T \to \infty$ the limiting distribution of $a_T^{-1}(M_T - b_T)$ exists.

Extreme Value Theory

Choose parameters $a_T > 0$ and $b_T \in \mathbb{R}$ such that when $T \to \infty$ the limiting distribution of $a_T^{-1}(M_T - b_T)$ exists.

The last expression has a limit if and only if

$$\lim_{T\to\infty}T\left(1-F\left(b_T+a_Ty\right)\right)=-\ln H_{\xi,\mu,\sigma}\left(y\right),$$

where $H_{\xi,\mu,\sigma}(x)$ is the generalized extreme value distribution (GEV) defined as

$$H_{\xi,\mu,\sigma}\left(x\right) = \begin{cases} \exp\left\{-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-1/\xi}\right\} & \xi \neq 0, \\ \exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\} & \xi = 0. \end{cases}$$
(1)

Here $(x)_+ = \max(x, 0)$, and $\xi, \mu \in \mathbb{R}$ and $\sigma > 0$ are the shape, location and scale parameter respectively.

For both the commodities $\{X_t^1\}_{t\geq 0}$ and currencies $\{X_t^2\}_{t\geq 0}$ a vector of random variables $\{(t_t^m, Y_t^m)\}_{t\geq 1}$ is obtained in which t_t^m characterizes the time of the *i*-th extreme event, and Y_t^m the mark, $Y_t^m = X_{t_t}^m - u^m$, with high threshold $u^m > 0$.



Here, $N^m(t)$, $t \in \mathbb{R}$, corresponds to the stochastic point process or left-continuous counting process describing the dynamic of occurrence of the stochastic process $\{(t^m_i, Y^m_i)\}_{i \ge 1}$ before time t.

The conditional intensity of this process is defined as

$$\lambda^{m}(t, y \mid \mathscr{H}_{t}) = \lambda_{g}^{m}(t \mid \mathscr{H}_{t}) g^{m}(y \mid \mathscr{H}_{t}, t)$$

where:

- $\lambda_g^m(t \mid \mathscr{H}_t)$ describes the intensity of occurrence of the extreme events
- $g^m(y \mid \mathscr{H}_t, t)$ the density function of the exceedances or marks
- The history of the process $\mathscr{H}_t = \{(t_i^m, Y_i^m); \forall (m, i) : t_i^m < t\}.$

The conditional intensity of this process is defined as

$$\lambda^{m}(t, y \mid \mathscr{H}_{t}) = \lambda_{g}^{m}(t \mid \mathscr{H}_{t}) g^{m}(y \mid \mathscr{H}_{t}, t)$$

where:

- $\lambda_q^m(t \mid \mathscr{H}_t)$ describes the intensity of occurrence of the extreme events
- $g^m(y \mid \mathcal{H}_t, t)$ the density function of the exceedances or marks
- The history of the process $\mathscr{H}_t = \{(t^m_i, Y^m_i); \forall (m, i) : t^m_i < t\}.$

The intensity of a multivariate Hawkes process is given by

$$\lambda_g^m(t \mid \mathscr{H}_t) = \mu_m + \sum_{k=1}^M \eta_{mk} \sum_{i:t_i^k < t} h_{mk}(y_i^k, t - t_i^k).$$

The exponential kernel $h_{mk}(y_i^k, t - t_i^k) = \alpha_k \exp(\delta_{mk}y_i^k - \alpha_k(t - t_i^k))$ represents the instant influence of events in series *k* on the intensity of *m*.

The probability density function of the marks is well approximated by the density function of Generalized Pareto Distribution (GPD)

$$g^{m}(y \mid \mathscr{H}_{t}, t) = \begin{cases} \frac{1}{\beta^{m}(y \mid \mathscr{H}_{t})} \left(1 + \xi^{m} \frac{y}{\beta^{m}(y \mid \mathscr{H}_{t})}\right)^{\frac{-1}{\xi^{m}} - 1}_{+} &, \xi^{m} \neq 0\\ \frac{1}{\beta^{m}(y \mid \mathscr{H}_{t})} \exp\left(\frac{-y}{\beta^{m}(y \mid \mathscr{H}_{t})}\right) &, \xi^{m} = 0 \end{cases}$$
(2)

with shape ξ^m and scale $\beta^m(y | \mathscr{H}_t)$ depending on the internal history of the process through the exponential kernel and aims to capture the impact of the size of the extreme events on their subsequent distribution

$$\beta^m(\mathbf{y} \mid \mathscr{H}_t) = \beta_0^m + \sum_{k=1}^M \beta_k^m \sum_{i: t_i^k < t} h_{mk}(\mathbf{y}_i^k, t - t_i^k)$$

The conditional intensity in the *m*-th dimension in the multivariate Hawkes-POT model, assuming $\xi^m \neq 0$ is given by

$$\lambda^{m}(t, y|\mathscr{H}_{t}) = \frac{\lambda_{g}^{m}(t|\mathscr{H}_{t})}{\beta^{m}(y|\mathscr{H}_{t})} \left(1 + \xi^{m} \frac{y}{\beta^{m}(y|\mathscr{H}_{t})}\right)^{\frac{1}{\xi^{m}} - 1}$$
(3)

Under this specification all the parameters with exception of the shape parameter ξ^m are restricted to be positive. Finally, the log-likelihood for such processes in a range (0,T] is given by:

$$\ln L = \sum_{m=1}^{M} \sum_{i=1}^{N^m(T)} \left\{ \ln g(y|\mathscr{H}_t) + \ln \lambda_g^m(t|\mathscr{H}_t) \right\} - \sum_{m=1}^{M} \int_0^T \lambda_g^m(s|\mathscr{H}_s) ds.$$
(4)

Multivariate DCC-GARCH-EVT

Stage 1: Introduced by Engle [2002]

• The conditional mean of X_t^m follows a AR (1) process

$$X_t^m = \phi^m + \theta^m X_{t-1}^m + \varepsilon_t^m,$$

where $\varepsilon_t^m = \zeta_t^m \sqrt{h_t^m}$ is an error term of the mean in each series and z_t^m is assumed to follow a multivariate *t*-Student distribution.

• The GJR-GARCH model [Glosten et al., 1993]

$$h_{t}^{m} = \omega^{m} + a^{m} \left(\varepsilon_{t-1}^{m}\right)^{2} + b^{m} h_{t-1}^{m} + \gamma^{m} \left(\varepsilon_{t-1}^{m}\right)_{+}^{2},$$

• The EGARCH model [Nelson, 1991]

$$\ln(h_{t}^{m}) = \omega^{m} + a^{m} \left[\frac{|\varepsilon_{t-1}^{m}|}{\sqrt{h_{t-1}^{m}}} - \sqrt{2/\pi} \right] + b^{m} \ln(h_{t-1}^{m}) + \gamma^{m} \left(\frac{\varepsilon_{t-1}^{m}}{\sqrt{h_{t-1}^{m}}} \right),$$

• The simple GARCH model [Bollerslev, 1986]

$$h_t^m = \omega^m + a^m \left(\varepsilon_{t-1}^m\right)^2 + b^m h_{t-1}^m$$

Multivariate DCC-GARCH-EVT

Stage 2: We apply the refinement proposed by McNeil and Frey [2000]. Specifically, the GPD defined in (2) is used to model the residuals z_t^m above a threshold $u^m > 0$, as follows

$$g^{m}(z_{t}^{m}-u^{m}) = g^{m}\left(\frac{\mathcal{E}_{t}^{m}}{\sqrt{h_{t}^{m}}}-u^{m}\right)$$
$$= g^{m}\left(\frac{X_{t}^{m}-\phi^{m}+\theta^{m}X_{t-1}^{m}-u^{m}\sqrt{h_{t}^{m}}}{\sqrt{h_{t}^{m}}}\right).$$

Stage 3: DCC-GARCH model introduced by Engle [2002]

The conditional covariance matrix \mathbf{H}_t can be decomposed into the terms of the a diagonal matrix $\mathbf{D}_t = \left\{ \text{diag}\left(\sqrt{h_t^1}, \sqrt{h_t^2}\right) \right\}$ and the a conditional correlation matrix \mathbf{R}_t , as follows

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t,$$

where $\mathbf{R}_t = \operatorname{diag}\left(\mathbf{Q}_t\right)^{-1/2} \mathbf{Q}_t \operatorname{diag}\left(\mathbf{Q}_t\right)^{-1/2}$ and the matrix \mathbf{Q}_t has the following specification

$$\mathbf{Q}_{t} = (1 - \pi_{1} - \pi_{2})\bar{\mathbf{Q}} + \pi_{1} \left(z_{t-1} z_{t-1} \right) + \pi_{2} \left(\mathbf{Q}_{t-1} \right),$$

with $z_{t-1} = \{z_{t-1}^1, z_{t-1}^2\}$ and $\bar{\mathbf{Q}}$ corresponding to the (2×2) unconditional covariance matrix of the standardised residuals. The coefficients, π_1 and π_2 are non-negative parameters, with $\pi_1 + \pi_2 < 1$, which implies that $\mathbf{H}_t > 0$.

Multivariate DBEKK-EVT

Stage 1: The DBEKK model was proposed by Engle et al. [1993]

• The conditional mean of X_t^m as an AR (1) process as follows

$$X_t^m = \phi^m + \theta^m X_{t-1}^m + \varepsilon_t^m,$$

where ϕ^m and θ^m are constant terms and $\varepsilon_t^m = z_t^m \sqrt{h_t^m}$ is an error term of the mean in each series and z_t^m is assumed to follow a multivariate *t*-Student distribution.

Stage 2: Following the structure of , the (2×2) conditional covariance matrix H_t corresponds to

$$\mathbf{H}_{t} = \mathbf{C}C' + \mathbf{A}\varepsilon_{t-1}\varepsilon_{t-1}A' + \mathbf{B}\mathbf{H}_{t-1}B',$$

where C is a (2×2) lower triangular matrix A and B are both (2×2) diagonal matrices.

Stage 3 : We apply the refinement proposed by McNeil and Frey [2000].

Evaluating Value-at-Risk

Value at Risk (VaR) represents the percentage loss that a portfolio will face over a predefined period of time, at a certain confidence level α is a popular measure of risk.

• The Hawkes-POT model

$$VaR_{\alpha}^{t+1} = u + \frac{\beta(y|\mathscr{H}_{t})}{\xi} \left(\left(\frac{1-\alpha}{\lambda_{g}(t+1|\mathscr{H}_{t})} \right)^{-\xi} - 1 \right),$$
(5)

where the superscript for dimension m has been removed for ease of exposition.

• The DCC-GARCH-EVT model

$$VaR_{\alpha}^{t+1} = \phi^m + \theta^m X_t^m + z_q^m \sqrt{h_t^m}$$
(6)

where z_q^m is the upper qth quantile of the marginal distribution of z_t which, by assumption, does not depend on *t*.

Model Specification (Hawkes-POT)

	Ground Process														
			Commodity						Currency						
	Model	μ_1	η_{11}	η_{12}	α_1	δ_{11}	δ_{12}	δ_{22}	δ_{21}	μ_2	η_{22}	η_{21}	α_2		
a	1	0.060	0.118	0.083	0.061	0.487	0.493	0.380	0.001	0.046	0.315	0.071	0.072		
		(0.014)	(0.116)	(0.081)	(0.042)	(0.091)	(0.170)	(0.053)	(0.003)	(0.010)	(0.098)	(0.013)	(0.021)		
tral	2	0.050	0.283		0.041	0.416		0.389	0.001	0.044	0.284	0.128	0.080		
SIL		(0.010)	(0.074)		(0.012)	(0.050)		(0.054)	(0.002)	(0.010)	(0.091)	(0.013)	(0.022)		
~	3	0.050	0.283		0.041	0.416		0.389	0.389	0.044	0.284	0.128	0.080		
		(0.010)	(0.074)		(0.012)	(0.050)		(0.054)	(0.054)	(0.010)	(0.091)	(0.013)	(0.022)		
B	1	0.057	0.136	0.066	0.046	0.406	0.750	1.305	0.077	0.049	0.157	0.133	0.055		
		(0.011)	(0.071)	(0.107)	(0.013)	(0.069)	(1.422)	(0.183)	(0.460)	(0.011)	(0.063)	(0.015)	(0.013)		
lad	2	0.060	0.174		0.044	0.389	0.074	1.304		0.049	0.157	0.136	0.054		
Car		(0.010)	(0.071)		(0.011)	(0.064)	(0.452)	(0.183)		(0.011)	(0.063)	(0.015)	(0.013)		
0	3	0.060	0.174		0.044	0.389		1.303		0.048	0.160	0.151	0.054		
		(0.010)	(0.071)		(0.011)	(0.064)		(0.181)		(0.010)	(0.060)	(0.012)	(0.012)		
				Generaliz	zed Pareto	Distribution	Function								
			Comr	nodity			Currency								
		β_0^1	β_1^1	β_2^1	ξ1	β_0^2	β_2^2	β_1^2	ξ ₂	Log-Lik	AIC				
	1	0.617	1.523	0.001	0.123	0.320	1.993	0.001	0.209	1950.822	-3861.644				
		(0.110)	(0.577)	(0.001)	(0.062)	(0.058)	(0.339)	(0.001)	(0.069)						
Australia	2	0.561	1.953		0.094	0.332	1.931	0.001	0.221	1954.675	-3875.349				
Australia		(0.091)	(0.442)		(0.063)	(0.057)	(0.324)	(0.001)	(0.068)						
	3	0.561	1.953		0.094	0.332	1.931		0.221	1954.675	-3879.350				
		(0.091)	(0.442)		(0.063)	(0.057)	(0.324)		(0.068)						
	1	0.870	1.564	0.001	0.156	0.178	0.881	0.001	0.186	1940.332	-3840.664				
		(0.116)	(0.588)	(0.001)	(0.044)	(0.033)	(0.230)	(0.001)	(0.046)						
Canada	2	0.853	1.698		0.150	0.178	0.883	0.001	0.187	1940.927	-3847.854				
		(0.114)	(0.587)		(0.044)	(0.033)	(0.231)	(0.001)	(0.046)						
	3	0.853	1.698		0.150	0.177	0.885		0.187	1940.984	-3851.968				

Table: Results of the Hawkes-POT models for Australia and Canada for the in-sample period. Negative logarithmic returns are applied to future prices of gold (subscript 1) and the Australian dollar (subscript 2), and Oil (subscript 1) and the Canadian dollar (subscript 2), respectively. Standard errors are in parentheses. Log-lik corresponds to the log-likelihood, while AIC corresponds to the value of the Akaike information criterion.

Model Specification (DCC-GARCH-EVT)

		AB(1) - GARCH(1.1)													
				Comm	odity		()		· · ·	Currency	Currency				
	Model	ϕ^1	θ^{1}	ω^1	a^1	b^1	γ^1	ϕ^2	θ^2	ω^2	a^2	b^2	γ^2		
	1	-4.E-04	0.003	2.E-06	0.051	0.938	0.002	6.E-05	-0.040	1.E-06	0.086	0.934	-0.059		
9		(2.E-04)	(0.026)	(9.E-06)	(0.085)	(0.088)	(0.021)	(1.E-04)	(0.021)	(1.E-06)	(0.014)	(0.017)	(0.017)		
B	2	-5.E-04	0.004	-0.134	0.008	0.984	0.128	8.E-05	-0.049	-0.086	0.045	0.991	0.143		
Ist		(3.E-04)	(0.025)	(0.007)	(0.003)	(0.000)	(0.031)	(1.E-04)	(0.022)	(0.012)	(0.011)	(0.001)	(0.025)		
4	3	-4.E-04	0.003	2.E-06	0.052	0.938		-5.E-05	-0.043	1.E-06	0.073	0.920			
		(2.E-04)	(0.026)	(8.E-06)	(0.072)	(0.080)		(1.E-04)	(0.021)	(5.E-06)	(0.085)	(0.085)			
	1	-3.E-05	-0.041	1.E-06	0.059	0.956	-0.033	1.E-05	-0.030	1.E-06	0.060	0.941	-0.016		
anada		(3.E-04)	(0.020)	(1.E-06)	(0.008)	(0.004)	(0.010)	(1.E-04)	(0.020)	(1.E-06)	(0.016)	(0.015)	(0.013)		
	2	-1.E-05	-0.033	-0.041	0.035	0.995	0.089	3.E-05	-0.032	-0.082	0.015	0.992	0.120		
		(3.E-04)	(0.020)	(0.002)	(0.007)	(0.000)	(0.005)	(1.E-04)	(0.019)	(0.015)	(0.012)	(0.001)	(0.032)		
0	3	-2.E-04	-0.042	1.E-06	0.045	0.952		-9.E-06	-0.030	1.E-06	0.053	0.940			
		(3.E-04)	(0.021)	(2.E-06)	(0.014)	(0.014)		(9.E-05)	(0.020)	(1.E-06)	(0.015)	(0.016)			
			DCC			E	VT								
			Joint		Comr	nodity	Currency								
		π_1	π_2	v	β_1	ξι	β_2	ξ2	Log-Lik	AIC					
	1	0.044	0.921	7.023	0.627	0.101	0.671	-0.076	16574.356	-33108.712					
Australia		(0.011)	(0.026)	(0.797)	(0.058)	(0.067)	(0.057)	(0.057)							
	2	0.043	0.926	7.042	0.615	0.103	0.664	-0.064	16571.499	-33102.999					
		(0.010)	(0.022)	(0.542)	(0.056)	(0.066)	(0.057)	(0.059)							
	3	0.044	0.921	6.894	0.624	0.104	0.697	-0.102	16561.672	-33087.345					
		(0.011)	(0.026)	(0.707)	(0.058)	(0.068)	(0.058)	(0.054)			-				
	1	0.021	0.975	13.142	0.555	0.006	0.649	-0.162	16628.098	-33216.197					
Canada		(0.006)	(0.008)	(1.941)	(0.047)	(0.058)	(0.052)	(0.053)							
	2	0.021	0.976	13.066	0.564	-0.009	0.671	-0.172	16618.021	-33196.043					
		(0.006)	(0.008)	(1.876)	(0.047)	(0.056)	(0.055)	(0.054)							
	3	0.019	0.978	13.017	0.593	-0.024	0.659	-0.174	16612.391	-33188.781					
		(0.006)	(0.008)	(1.955)	(0.049)	(0.054)	(0.053)	(0.052)							

Table: Results of the DCC-GARCH-EVT models for Australia and Canada in the period In-Sample. Negative logarithmic returns are applied to future prices of gold (subscript 1) and the Australian dollar (subscript 2), and Oil (subscript 1) and the Canadian dollar (subscript 2), respectively. Standard errors are in parentheses. Log-lik corresponds to the log-likelihood, while AIC corresponds to the value of the Akaike information criterion.

Intensity vs Volatility



Intensity vs Volatility



Value at Risk (In-Sample)

				Australia						Canada					
			α level	Excep	LR_{uc}	LRind	LR_{cc}	DQ_{hit}	Excep	LR_{uc}	LRind	LR_{cc}	DQ_{hit}		
	Model 3	Commodity	0.95	132	0.59	0.42	0.62	0.43	129	0.96	0.17	0.40	0.19		
5			0.99	30	0.35	0.37	0.43	0.37	23	0.59	0.52	0.70	0.52		
Ă			0.999	7	0.02	0.84	0.07	0.84	5	0.18	0.89	0.40	0.89		
sey		Currency	0.95	127	0.93	0.16	0.36	0.17	126	0.82	0.73	0.92	0.74		
1×E			0.99	29	0.46	0.34	0.49	0.35	28	0.65	0.31	0.54	0.31		
Ï			0.999	10	0.00	0.78	0.00	0.78	10	0.00	0.78	0.00	0.78		
F	Model 1	Commodity	0.95	61	0.00	0.25	0.00	0.27	116	0.24	0.44	0.38	0.45		
Ψ			0.99	9	0.00	0.02	0.00	0.02	21	0.33	0.56	0.53	0.56		
F			0.999	1	0.27	0.98	0.55	0.98	2	0.71	0.96	0.93	0.96		
Ť		Currency	0.95	114	0.25	0.41	0.37	0.42	180	0.00	0.64	0.00	0.63		
ġ			0.99	29	0.47	0.34	0.49	0.35	66	0.00	0.55	0.00	0.56		
8			0.999	6	0.06	0.87	0.18	0.87	13	0.00	0.72	0.00	0.72		
Ō															
	Model 1	Commodity	0.95	58	0.00	0.20	0.00	0.21	118	0.33	0.06	0.11	0.07		
F			0.99	9	0.00	0.02	0.00	0.02	24	0.73	0.50	0.75	0.50		
μ			0.999	1	0.27	0.98	0.55	0.98	3	0.80	0.93	0.96	0.93		
ž		Currency	0.95	99	0.01	0.30	0.02	0.31	139	0.36	0.86	0.64	0.85		
BE			0.99	25	0.95	0.25	0.51	0.25	40	0.01	0.65	0.03	0.66		
ō			0.999	8	0.01	0.82	0.02	0.82	8	0.01	0.82	0.03	0.82		

Table: VaR accuracy tests for the in-sample period. Entries in the rows are the significance levels (p-values) of the respective tests, with the exception of the confidence level α for the VaR and the number of exceptions (Excep).

Value at Risk (forecasting)

							Canada						
			α level	Excep	LR_{uc}	LRind	LR_{cc}	DQ_{hit}	Excep	LR_{uc}	LRind	LR_{cc}	DQ_{hit}
ы	Model 3	Commodity	0.95	33	0.14	0.03	0.04	0.04	34	0.10	0.33	0.16	0.33
			0.99	7	0.42	0.68	0.66	0.68	9	0.11	0.59	0.25	0.57
ď,			0.999	1	0.54	0.95	0.83	0.95	4	0.00	0.83	0.01	0.80
sey		Currency	0.95	31	0.27	0.48	0.42	0.46	36	0.04	0.73	0.12	0.74
A.			0.99	6	0.69	0.05	0.14	0.05	10	0.05	0.01	0.01	0.01
Ĩ			0.999	0	0.31	1.00	0.60	1.00	3	0.02	0.85	0.06	0.85
F	Model 1	Commodity	0.95	9	0.00	0.57	0.00	0.58	24	0.68	0.13	0.29	0.14
Ξ			0.99	2	0.11	0.90	0.27	0.90	3	0.29	0.85	0.56	0.85
Ε			0.999	0	0.31	1.00	0.59	1.00	0	0.31	1.00	0.59	1.00
R		Currency	0.95	28	0.70	0.64	0.83	0.65	37	0.04	0.66	0.11	0.67
Ρg			0.99	5	0.93	0.76	0.95	0.76	11	0.03	0.22	0.04	0.22
ŭ			0.999	1	0.56	0.95	0.84	0.95	2	0.12	0.90	0.29	0.90
Ř													
	Model 1	Commodity	0.95	10	0.00	0.53	0.00	0.54	28	0.71	0.68	0.86	0.69
F			0.99	2	0.11	0.90	0.27	0.90	5	0.92	0.76	0.95	0.76
щ			0.999	0	0.31	1.00	0.59	1.00	1	0.56	0.95	0.84	0.95
В.		Currency	0.95	26	0.99	0.77	0.96	0.78	30	0.44	0.83	0.73	0.83
Ë		-	0.99	4	0.58	0.80	0.83	0.80	11	0.03	0.22	0.04	0.22
DB			0.999	1	0.56	0.95	0.84	0.95	4	0.00	0.02	0.00	0.02

Table: VaR accuracy tests for the out-sample period. Entries in the rows are the significance levels (p-values) of the respective tests, with the exception of the confidence level α for the VaR and the number of exceptions (Excep).

Value at Risk



Figure: VaR estimation to 99% with black line for the Hawkes-POT, DCC-GARCH-EVT and DBEKK-EVT, applied to the negative log returns of the future gold prices (Top panel) and the Australian dollar (Below panel). The vertical dotted line indicates the division between the in-sample period from 4 January 2005 to 31 December 2014, and the backtesting period from 1 January 2015 until 31 December 2016. The dashed line in the graphic represents the beginning of the backtesting period.

Value at Risk



Figure: VaR estimation to 99% with black line for the Hawkes-POT, DCC-GARCH-EVT and DBEKK-EVT applied to the negative log returns of the future oil prices (Top panel) and the Canadian dollar (Below panel). The vertical dotted line indicates the division between the in-sample period from 4 January 2005 to 31 December 2014, and the backtesting period from 1 January 2015 until 31 December 2016. The dashed line in the graphic represents the beginning of the backtesting period.

Conclusions

- We examine dependence at extreme levels in the relationship between two well-known commodity currencies.
- We consider two perspectives based on extreme value theory, with a multivariate Hawkes-POT marked point process framework, based on intensity of extreme events and the DCC-GARCH-EVT and DBEKK-EVT models based on correlations and volatility.
- Overall, both the intensity and volatility based approaches can be considered to be complementary.
- The benefit of the Hawkes point process approach is that it captures the causal unidirectional dependence between both markets and a feedback between the intensity and magnitude of extreme events.
- While point process models have often found to provide superior predictions in an univariate setting, the multivariate volatility models provides forecasts of similar quality in the backtesting period.

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