Modeling Extreme Risks in Commodities and Commodity Currencies

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Abstract

This paper analyzes extreme co-movements between the Australian and Canadian commodity currencies, and gold and oil markets respectively. We propose two perspectives based on extreme value theory to analyze the behavior of these markets in periods of financial crisis. On the one hand, the intensity of the extreme events is represented by self-excited marked point processes with a multivariate extension of the Hawkes-POT model, while contemporaneous co-movements are characterized utilizing a DCC-GARCH-EVT model, following a more traditional approach. We find that both stochastic processes, intensity and volatility, follow similar patterns throughout the analyzed period. The main advantage of the proposed Hawkes-POT model is that it captures the unidirectional influence of the commodity on the currency, consistent with previous literature. Hawkes-POT model allows slightly better accuracy value at risk results in the in-sample period. While, the results are mixed in the backtesting period.

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1. Introduction

In many countries there are principal commodities that dominate a significant share of total exports. The prices at which these commodities are traded are crucial to the economic performance and generally have a significant effect on exchange rates behavior (Edwards, 1986). Because of the importance of this relationship these currencies are often called "commodity currencies". Previous studies have analyzed the relationship between commodity prices and exchange rates, using commonly cointegration and error correction models (Hatzinikolaou and Polasek, 2005; Wu, 2013; Kohlscheen, 2010; Bjørnland and Hungnes, 2005), and causality models (Chen et al., 2010; Choudhri and Schembri, 2014; Chan et al., 2011). Overall, it is possible to determine that the relationship between commodities and currencies exist (Cashin et al., 2004; Chen and Rogoff, 2003; Bowman et al., 2005; Bodart et al., 2012; Kato et al., 2012). Nevertheless, despite the extensive literature around these currencies, there is little understanding of their behavior at extreme levels, allowing a deeper analysis of the interaction of these markets in periods of large losses. Several research have established important changes in the behavior of time series during periods of stress (Stöber and Czado, 2014), particularly, in the returns of the currency and the exchange rate, large movements are generated by great recessions Ready et al. (2017).

The underlying relationship between extreme events and comovements during financial turmoil periods is of growing interest among investors and researchers (Belhajjam et al., 2017). However, to analyze the complex behavior of these markets at extreme levels is not a straightforward task, mainly because it involves capturing stylized facts present in financial series, such as clustering of extreme events and heavy tails (Hung et al., 2008; Liu and Tang, 2011; Delatte and Lopez, 2013; Aboura and Chevallier, 2015). A large literature presents tools to measure risk in periods of stability, but these are not valid in turbulent times (Massacci, 2016). A drawback of the commonly used models is the potential misestimation of the tails of the probability distribution (Kellner and Gatzert, 2013). Consequently it is essential to incorporate methodologies that allow to explain appropriately the complex relationship of these markets in extreme economic conditions. To this end, a multivariate extension of the Hawkes-Peaks over Threshold (POT) model of Chavez-Demoulin et al. (2005) and Chavez-Demoulin and Davison (2012) is proposed. This methodology, based on self-exciting point processes and Extreme Value Theory (EVT) permits the analysis of extreme events which are not necessarily iid, capturing the intrinsic characteristics of the return series during periods of crisis. Additionally for a broader perspective, we propose to analyze the behavior of extreme values following a more traditional approach. We combine the DCC-GARCH dynamical conditional correlation model proposed by Engle and Sheppard (2001) with a refinement of EVT extending the univariate conditional model

proposed by McNeil and Frey (2000) to the multivariate case. This methodology allows model the cluster behavior of extreme events caused by the stochastic volatility and to capture the dynamic of the conditional correlations among markets.

The empirical evidence has focused on examining particularly the currencies of Australia, Canada and New Zealand (Sanidas, 2014; Ready et al., 2017). Following this line of research, we consider the Australian and Canadian market of which we have at disposal a longer database, necessary for analysis of the extreme event. The contribution of this paper is twofold. On the one hand, we propose a nobel multivariate extension of the Hawkes-POT model and a multivariate DCC-GARCH-EVT specification. On the other hand, in relation to empirical analysis, this paper contributes to the extensive literature of these well-known commodity currencies, from a perspective focused only on bear markets.

The main research questions posed are: Is it possible to explain the dynamics of extreme events in both currency and commodity returns based on their historical behavior? Can improved predictions of extreme risk be generated? Which stochastic process plays a more important role in predicting extreme events, intensity or volatility? Are both approaches complementary?

The methodologies proposed here along with the empirical results will provide participants in the commodity and currency markets, commodity traders, producers and wholesale consumers, a deeper understanding of the extreme risks they face. The main conclusion is that both approaches complement each other to characterize the relationship between currency and commodity. Considering the history of past extreme events it is possible to explain from two different perspectives, intensity and volatility, the dynamic behavior and the interaction of both markets at extreme levels. With the DCC-GARCH-EVT methodology we find a relationship between the volatility and correlation, although contemporaneously. While using the Hawkes-POT methodology it is possible to analyze the cross-excitation of extreme events, obtaining a feedback of the relationship between its intensity and magnitude. Causality (en el sentido de) allows to demonstrate at extreme levels that both currencies are commodity currencies. In the Australian and Canadian cases, the intensity in the currency is influenced mainly by the rate of occurrence of the extreme events in the commodity and not by the magnitude of these events.

In relation to the estimation and accuracy of the Value at Risk (VaR), slightly better results in-sample periods are obtained with Hawkes-POT models, whereas in the backtesting period the results are mixed, for which it is not possible to establish if the intensity or the volatility plays a more important role in the prediction of the extreme events. In the Australian case the best model is the Hawkes-POT, and in the Canadian case the best model is the DCC-GARCH-EVT.
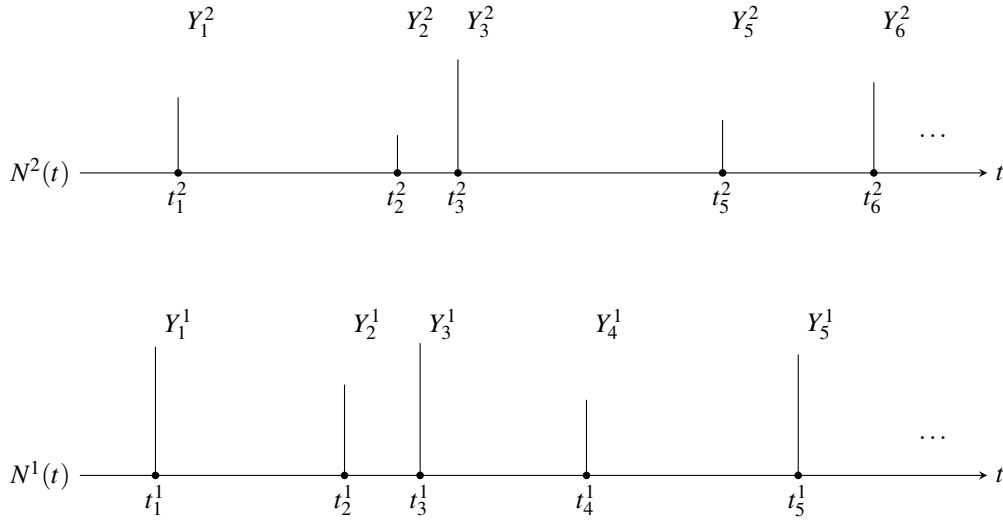
Figure 1: The Peaks over threshold procedure. The model considers the time of occurrence t_i of an extreme event i, where Y_i^m denotes the mark.

This paper is organized as follows. In Section 2, we introduce the multivariate Hawkes-POT model and the DCC-GARCH-EVT approach. Section 3 includes the empirical analysis, description of the models, estimation results and analysis of the risk measures. Finally, Section 4 presents the conclusions, study limitations and recommendations for future research.

2. Methodology

2.1. Multivariate Hawkes-POT

Here, the perspective of an investor concerned with losses is taken, therefore all subsequent analysis is based on the negative returns for both the commodities $\{X_t^1\}_{t \geq 0}$ and currencies $\{X_t^2\}_{t \geq 0}$. A popular method to model extreme events is the Peaks over Threshold (POT) methodology, with all values that are above a sufficiently high threshold considered to be extreme events. Figure 1 gives a graphical description of this procedure for both markets. Under this framework, a vector of random variables $\{(t_i^m, Y_i^m)\}_{i \geq 1}$ is obtained in which the superscript $m = 1, \ldots, M$ represents the m-th dimension of the model, t_i^m characterizes the time of the i-th extreme event, and Y_i^m characterizes the mark, $Y_i^m = X_{t_i}^m - u^m$, with high threshold $u^m > 0$. Here, $N^m(t), t \in \mathbb{R}$, corresponds to the stochastic point process or left-continuous counting process describing the dynamic of occurrence of the stochastic process $\{(t_i^m, Y_i^m)\}_{i \geq 1}$ before time t.

In this paper we propose to model the stochastic process $N^m(t)$ by means of an extension of an univariate self-exciting marked point process that capture the tendency of clustering in extreme events, the Hawkes-POT model (Chavez-Demoulin and McGill, 2012). The conditional intensity of this process is defined as

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$$\lambda^m(t, y \mid \mathcal{H}_t) = \lambda_g^m(t \mid \mathcal{H}_t) g^m(y \mid \mathcal{H}_t, t)$$

where $\lambda_g^m(t \mid \mathcal{H}_t)$ describes the intensity of occurrence of the extreme events, the self-exciting ground process, and $g^m(y \mid \mathcal{H}_t, t)$ the density function of the exceedances or marks, conditional on the history of the process $\mathcal{H}_t = \{(t_i^m, Y_i^m); \forall(m, i) : t_i^m < t\}$. According to the Pickands-Balkema-De Haan theorem (Balkema and de Haan, 1974; Pickands, 1975) the probability density function of the marks is well approximated by the density function of Generalized Pareto Distribution (GPD)

$$g^m(y \mid \mathcal{H}_t, t) = \begin{cases} \frac{1}{\beta^m}\left(1 + \xi^m \frac{y}{\beta^m}\right)_+^{\frac{-1}{\xi^m} - 1} & , \xi^m \neq 0 \\ \frac{1}{\beta^m} \exp\left(\frac{-y}{\beta^m}\right) & , \xi^m = 0 \end{cases} \tag{1}$$

with scale and shape parameters, β^m and ξ^m, respectively, where $(f)_+ = \max\{0, f\}$ defines the positive part of the function. In a multivariate Hawkes process, an extreme event in the dimension or series m, increases the likelihood of future events of this type within the same dimension (self-excitation) and also in other dimensions (cross-excitation), a commonly observed pattern in financial markets. The intensity of a multivariate Hawkes process is given by

$$\lambda_g^m(t \mid \mathcal{H}_t) = \mu_m + \sum_{k=1}^{M} \eta_{mk} \sum_{i:t_i^k < t} h_{mk}(y_i^k, t - t_i^k)$$

where μ_m corresponds to the immigrant rate of extreme events in dimension m that occur independent of self-excitation and η_{mk} determines the influence of events in dimension k on the occurrence of extreme events in the dimension m **(incluir causalidad)**. The exponential kernel $h_{mk}(y_i^k, t - t_i^k) = \alpha_k \exp(\delta_{mk} y_i^k - \alpha_k(t - t_i^k))$ represents the instant influence of events in series k on the intensity of m. Here, α_k is the rate of decay in the intensity from events in series k and δ_{mk} captures the impact of the size of events in series k on the intensity of m. When $m = k$ this is a self-exciting effect, otherwise it represents cross-excitation. The density function of the marks $g^m(y \mid \mathcal{H}_t, t)$ is conditional. Its scale parameter, $\beta^m(y \mid \mathcal{H}_t)$, is dependent on the internal history of the process through the exponential kernel and aims to capture the impact of the size of the extreme events on their subsequent distribution

$$\beta^m(y \mid \mathcal{H}_t) = \beta_0^m + \sum_{k=1}^{M} \beta_k^m \sum_{i:t_i^k < t} h_{mk}(y_i^k, t - t_i^k)$$

The conditional intensity in the m-th dimension in the multivariate Hawkes-POT model, assuming $\xi^m \neq 0$ is given by

$$\lambda^m(t, y \mid \mathcal{H}_t) = \frac{\lambda_g^m(t \mid \mathcal{H}_t)}{\beta^m(y \mid \mathcal{H}_t)}\left(1 + \xi^m \frac{y}{\beta^m(y \mid \mathcal{H}_t)}\right)^{\frac{1}{\xi^m} - 1} \tag{2}$$

Under this specification all the parameters with exception of the shape parameter ξ^m are restricted to be positive. Finally, the log-likelihood for such processes in a range $(0, T]$ is given by:

$$\ln L = \sum_{m=1}^{M} \sum_{i=1}^{N^m(T)} \left\{ \ln g(y|\mathcal{H}_t) + \ln \lambda_g^m(t|\mathcal{H}_t) \right\} - \sum_{m=1}^{M} \int_0^T \lambda_g^m(s|\mathcal{H}_s) ds. \tag{3}$$

In its general form, this model for M dimensions has a rich structure due to its flexibility in permitting different forms of dependence. However, this flexibility is associated with a high cost in the number of parameters. For this reason, in Section 3, restricted alternatives of this general model are also used to explain the relationship between commodities and currencies, and highlight links between extreme risks in the commodity and currency markets.

2.2. Multivariate DCC-GARCH-EVT

The multivariate DCC-GARCH-EVT approach consists in three main steps. We use a AR-GARCH specification to model the conditional mean and volatility of each return series. Then, by means of the innovations of the residuals we obtain the dynamic correlation between the markets and by applying EVT to the these we obtain risk measures at high levels. Again, we consider $\left\{ X_t^1 \right\}_{t \geq 0}$ and $\left\{ X_t^2 \right\}_{t \geq 0}$ as the vectors of the negative returns of the commodity and the currency, respectively. We model the conditional mean of X_t^m as an AR (1) process as follows

$$X_t^m = \phi^m + \theta^m X_{t-1}^m + \varepsilon_t^m,$$

where ϕ^m and θ^m are constant terms and $\varepsilon_t^m = z_t^m \sqrt{h_t^m}$ is an error term of the mean in each series. The latter is composed by the innovations vector of random variables iid z_t^m, for which we assume the multivariate t-Student distribution. In order to model the marginal conditional variances h_t^m we propose the following three specifications. The GJR-GARCH model (Glosten et al., 1993)

$$h_t^m = \omega^m + a^m \left(\varepsilon_{t-1}^m \right)^2 + b^m h_{t-1}^m + \gamma^m \left(\varepsilon_{t-1}^m \right)_+^2,$$

where $(f)_+ = max\{0, f\}$ defines the positive part of the function. The EGARCH model (Nelson, 1991)

$$\ln(h_t^m) = \omega^m + a^m \left[\frac{|\varepsilon_{t-1}^m|}{\sqrt{h_{t-1}^m}} - \sqrt{2/\pi} \right] + b^m \ln(h_{t-1}^m) + \gamma^m \left(\frac{\varepsilon_{t-1}^m}{\sqrt{h_{t-1}^m}} \right),$$

and the simple GARCH model (Bollerslev, 1986)

$$h_t^m = \omega^m + a^m \left(\varepsilon_{t-1}^m \right)^2 + b^m h_{t-1}^m$$

In addition, to capture the dynamic of the extreme events we apply the refinement proposed by McNeil and Frey (2000). Specifically, we use the GPD defined in (1) to model the residuals z_t^m above a threshold $u^m > 0$, as follows

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$$g^m\left(z_t^m - u^m\right) = g^m\left(\frac{\varepsilon_t^m}{\sqrt{h_t^m}} - u^m\right)$$

$$= g^m\left(\frac{X_t^m - \phi^m + \theta^m X_{t-1}^m - u^m \sqrt{h_t^m}}{\sqrt{h_t^m}}\right).$$

In order to represent the dependence between both markets, we propose the DCC-GARCH model introduced by Engle (2002), in this way the conditional covariance matrix $\mathbf{H_t}$ can be decomposed into the terms of the diagonal matrix $\mathbf{D_t} = \left\{diag\left(\sqrt{h_t^1}, \sqrt{h_t^2}\right)\right\}$ and the conditional correlation matrix R_t, as follows

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t,$$

$$\mathbf{H_t} = (1 - \pi_1 - \pi_2)\mathbf{H_0} + \pi_1\left(\varepsilon_{\mathbf{t-1}}\varepsilon_{\mathbf{t-1}}^{'}\right) + \pi_2\left(\mathbf{H_{t-1}}\right),$$

with $\varepsilon_{\mathbf{t-1}} = \left\{\varepsilon_{t-1}^1, \varepsilon_{t-1}^2\right\}$ and $\mathbf{H_0} = \left\{\omega^1/(1 - a^1 - b^1), \omega^2/(1 - a^2 - b^2)\right\}$ corresponds to the unconditional variance. On the other hand, π_1 and π_2 are non-negative scaling parameters capturing the shock and previous dynamic conditional correlations, with $\pi_1 + \pi_2 < 1$, which implies that $\mathbf{H}_t > 0$.[1]

We can perform the estimation of the parameters maximizing the log-likelihood function

$$\ln L = \sum_{t=1}^{T}\left\{\ln\left[\frac{\Gamma\left(\frac{\nu+T}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\right] - \frac{T}{2}\ln\left[\pi(\nu - 2)\right] + \frac{1}{2}\ln\left[\|\mathbf{H_t}(\theta)\|\right] - \frac{\nu+T}{2}\ln\left[1 + \frac{\varepsilon_{\mathbf{t}}^{'}(\theta)\mathbf{H_t}^{-1}\varepsilon_{\mathbf{t}}(\theta)}{\nu - 2}\right]\right\} \quad (4)$$

where θ is the vector of parameters of the DCC-GARCH model and ν is degree of freedom of the multivariate t-Student distribution function.

2.3. Evaluating Value-at-Risk

An estimate of Value at Risk (VaR) represents the percentage loss that a portfolio will face over a predefined period of time, at a certain confidence level α. The VaR is estimated in the period insample and backtesting for the returns in both markets. For Hawkes-POT model, it can be directly obtained from the intensity of the ground process and the parameters of the GPD for the size of the events, given by:

$$VaR_\alpha^{t+1} = u + \frac{\beta(y|\mathcal{H}_t)}{\xi}\left(\left(\frac{1-\alpha}{\lambda_g(t|\mathcal{H}_t)}\right)^{-\xi} - 1\right), \quad (5)$$

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[1]Note that when $\pi_1 = \pi_2 = 0$, $\mathbf{H}_t$ it is equivalent to CCC.

where the superscript for dimension *m* has been removed for ease of exposition.[2]

In the case of DCC-GARCH-EVT model, it can be obtained in terms of the conditional mean, denoted as:

$$VaR_\alpha^{t+1} = \phi^m + \theta^m X_t^m + z_q^m \sqrt{h_t^m} \tag{6}$$

where z_q^m is the upper qth quantile of the marginal distribution of z_t which, by assumption, does not depend on t.

Four widely used statistical tests are performed to evaluate the accuracy of the *VaR* from both models, with three of these tests based on the likelihood ratio tests of Christoffersen (1998). The first is the likelihood ratio unconditional coverage test (LR_{uc}) test which determines whether the number of exceptions, $I_t = I(X_t < -VaR_\alpha^t)$ (when the *VaR* prediction is breached) differ from its expected value at a given confidence level α. The second is a test of independence (LR_{ind}) that tests whether VaR exceptions are independent through time. The third statistic is the likelihood ratio test of conditional coverage (LR_{cc}), which simultaneously checks for independence and coverage. Finally, the dynamic quantile test (DQ_{hit}) of Engle and Manganelli (2004) is applied, which also tests for the presence of any dependence between the hits by defining $Hit_t = I_t - \alpha$, (therefore $E[Hit_t] = 0$). Then, the model to be estimated is $Hit_{t+1} = a + bHit_t + e_t$, and the null hypothesis $H_0 = a = b = 0$ tested (For details see AppendixB). Tests in the subsequent empirical analysis are undertaken at three levels of confidence: 0.95, 0.99, and 0.999.

3. Empirical Analysis

The empirical analysis focuses on Australia and Canada in terms of the co-movements in extreme risks between the exchange rates of these countries and the future prices of gold and Brent oil, respectively. Canada is the only country in the G-7 that is a net exporter of crude oil; it exports nearly 100% of its oil to the United States, the main consumer of the commodity (Bashar et al., 2013), with Australia the second-largest gold producer worldwide.

3.1. Data Description and Model Specifications

The financial series are daily observations obtained from Bloomberg. We analyze the losses, therefore, the negative logarithmic returns of the financial price series $X_t = -\ln(P_t/P_{t-1})$ are utilized. In

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[2]See Chavez-Demoulin and McGill (2012) for details pertaining to the derivation of this risk measure under the Hawkes-POT model.

order to determine the predictive ability of the model, the database was divided into two periods. The first 10 years are used for estimation and model fit, with observations ranging from 4 January 2005 to 31 December 2014. For the backtesting sample, the period from 1 January 2015 until 31 December 2016 is used. Table A.1 presents the descriptive statistics for each of the series. The results show the existence of common stylized facts, such as an asymmetry in the losses and heavy tails. Jarque-Bera and Box-Pierce tests show that the returns are not normally distributed and auto-correlation is present in the returns. In addition, LM-test indicates volatility in both markets.

The multivariate Hawkes-POT model is specified in (2). In this model it is possible to characterize the stochastic process of the intensity and the magnitude or size of the extreme events. In order to analyze the dependency this approach considers 6 parameters that allow cross-excitement through their intensities (η_{12}, η_{21}), as well as, through the magnitudes of the extreme returns $(\beta_2^1, \delta_{12}, \beta_1^2, \delta_{21})$. A general model and two restricted versions are proposed to examine the nature of the links in extreme risk between the commodity and currency markets. Model 1, is the most general specification and forms a benchmark to the restricted model will be compared. Consistent with previous studies in the literature (Cashin et al., 2004; Chen and Rogoff, 2003; Bowman et al., 2005; Bodart et al., 2012; Kato et al., 2012), Model 2 and Model 3 are unidirectional specifications that characterize dependency structures, where the currency is influenced by the commodity. Model 2 fully restricts the influence from the currency market to the commodity market but still retains all of the links from the commodity to the currency market (i.e., $\eta_{12} = \beta_2^1 = \delta_{12} = 0$). Finally, in addition to the Model 2 constraints, Model 3 allows only the influence of the intensity of the extreme events in the currency (i.e., $\eta_{12} = \beta_2^1 = \delta_{12} = \beta_1^2 = \delta_{21} = 0$). In this way we determine whether to capture the dynamic behavior of the extreme events in the currency, we must focus on the intensity or the magnitude of the extreme events in the commodity.

The second approach corresponds to the multivariate DCC-GARCH-EVT model. Both for the Australian case and for the Canadian case we try different alternatives to determine the number of lags of the conditional mean and variance to be included in the model. In addition, we analyzed the existence of volatility asymmetry with the Sign of Bias Test, the Negative and Positive Size Test and the Joint Test, following the study proposed by Engle and Ng (1993). The model AR(1)-GARCH(1,1) is the option that best fits in these markets, coinciding with a wide variety of data series (Bauwens et al., 2012). Three specifications are proposed for modeling the conditional variances, GJR-GARCH, EGARCH and GARCH models, which correspond to Model 1, Model 2 and Model 3, respectively. Finally, in order to decide whether to use the DCC model, we apply the CCC test of Engle and Sheppard (2001). In both cases the DCC specification is prefered.

To determine the fraction of observations in the tail of the distribution, we have to determine an optimal threshold. The choice of this threshold is subject to a balance between bias and variance. That is, increasing the sample size of extreme events will bias the approximation through the GPD of the tail's distribution, but the variance of the estimates will be reduced. While decreasing this proportion increases the variance of the estimators, unbiased parameters are obtained. Most techniques for selecting the threshold are based on graphical methods and bootstrap techniques (McNeil et al., 2005; Scarrott and MacDonald, 2012; Chavez-Demoulin and Davison, 2012). However, these techniques can be subjective in their interpretation. Similar to Herrera (2013), this research proposes a pragmatic way to determine the threshold. The idea is to choose the threshold for which allows us to obtain the most accurate tests for the in-sample VaR (LR_{uc}, LR_{ind}, LR_{cc} and DQ_{hit}). It is expected that this method will allow us to obtain better results in backtesting analysis than others might. To this end, we consider the thresholds of the 0.90 to 0.94 quantiles, counting the number of approved statistical tests for each scenario. According to these estimates, the threshold that maximizes the accuracy of the in-sample VaR is the 0.90 quantile, as it is also suggested by McNeil and Frey (2000). The models are estimated by maximizing the log-likelihoods (3) and (4).

3.2. Relationship between extreme price intensity and volatility in commodity currencies

Engle and Russell (1998) was the first in investigate the close relationship between intensity and volatility in a high frequency framework. In this section, we extend these results to the context extreme price intensity and its relationship with volatility in commodity currencies. One of the main features of our approach is that while volatility is defined using all returns, the intensity of extreme events is only defined by those that have exceeded a high threshold. Both approaches have their advantages and disadvantages, but analyzing together they give a clear picture of the dynamics of this type of events.

Table A.3 reports the estimation results for first competitive specification, the Hawkes-POT models, for the Australian and Canadian markets. The results reveals the importance of the cross-sectional dependence between the commodity and the currency in terms of feedback between intensities and magnitudes. Regarding the model specification, in both cases Model 3 is superior in terms of Akaike information criterion (AIC). It means that the intensity the extreme events of the currency are influenced by the occurrence rate of the events in the commodity, although apparently the magnitude of the extreme event is not so relevant to characterize the behavior of the commodity currencies.

An important feature of the Hawkes-POT model is that it allows one to distinguish the proportion of extreme events that are due to exogenous events, represented by the immigration rates (μ_1 and
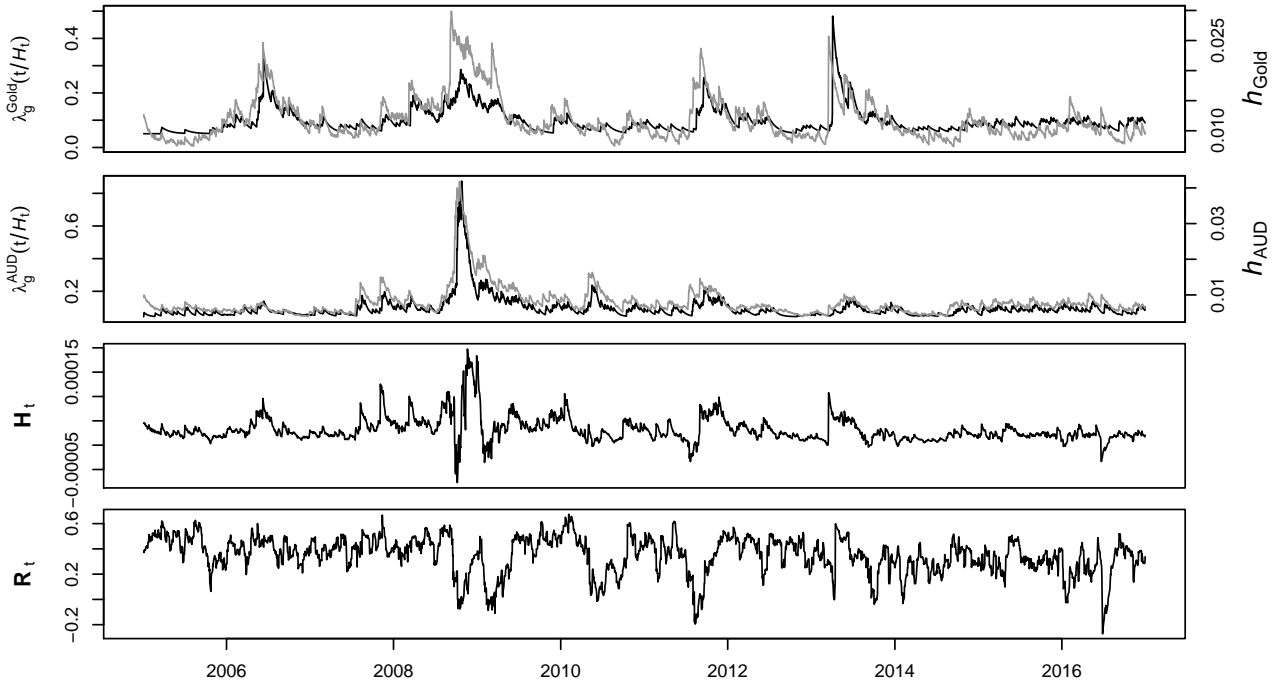
Figure 2: Conditional intensity (Black line) and conditional variance (Gray line) are shown in the top two panels, for gold and Australian dollar respectively. Correlation ($\mathbf{H}_t$) and covariance ($\mathbf{R}_t$) between both markets are presented in the two lower panels.

μ_2), and the proportion of extreme events that are exclusively due to self-excitation (η_{11} and η_{22}) and cross-excitation (η_{12} and η_{21}) (Hardiman et al., 2013). Overall, low immigrant rates are found along with higher values of the branching coefficients. In the Australian case, the immigration rate is higher ($\mu_1 = 0.050$) for gold than for the Australian dollar ($\mu_2 = 0.044$), consistent with the high sensitivity of commodities to external shocks. The self-exciting coefficients, which corresponds to the probability that extreme events occur only by effects of the same market, are almost identical ($\eta_{11} = 0.283$ and $\eta_{22} = 0.284$). In addition, the cross-excitation coefficient, which indicates the proportion of extreme events in the Australian dollar that are caused by extreme losses occurring in the returns of gold, also shows evidence of cross-excitement ($\eta_{21} = 0.128$).

Similarly for Canada, the immigration rate is higher ($\mu_1 = 0.060$) for oil than for the Canadian dollar ($\mu_2 = 0.048$) and the rate of self-excitation is lower in the Canadian dollar ($\eta_{22} = 0.160$) than for the oil ($\eta_{11} = 0.174$). In a particular way, the cross-excitation coefficient is highly representative within the fit ($\eta_{21} = 0.151$), so the rate of occurrence of extreme events in oil strongly influences the intensity in the currency, which is mainly due to the global commodity characteristics of oil (Chan et al., 2011; Ferraro et al., 2015). In the other hand, the coefficient of the decay kernel function, these are stronger for the currency in both cases (Australia: $\alpha_2 = 0.080$, Canada: $\alpha_2 = 0.054$), suggesting that after an extreme event the currency would achieve stability in the market more quickly. Further, the parameters δ_{11} and δ_{22} are also statistically significant in both countries, indicating that the magnitude of past
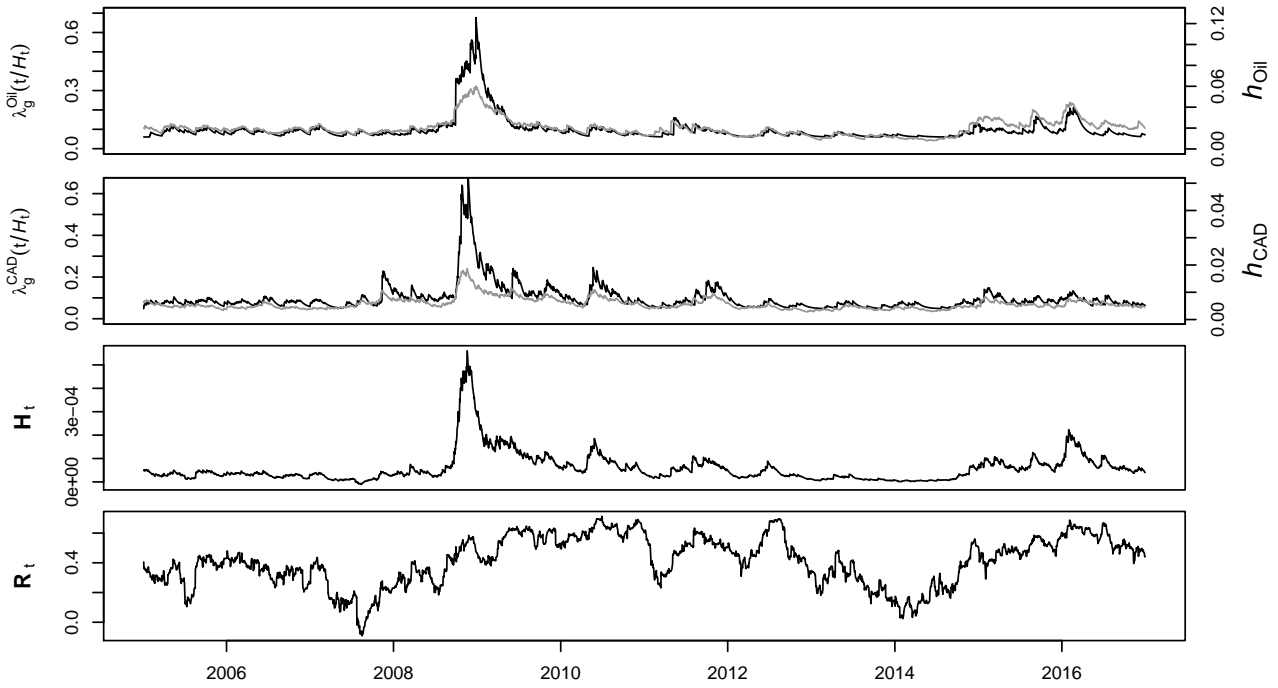
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Figure 3: Conditional intensity (Black line) and conditional variance (Gray line) are shown in the top two panels, for oil and Canadian dollar respectively. Correlation ($\mathbf{H_t}$) and covariance ($\mathbf{R_t}$) between both markets are presented in the two lower panels

events influence the intensity of future extreme events on the same market.

In relation to the dynamic of the stochastic process of the marks, the coefficients associated to the influence of the intensity on the magnitude of the marks in both markets show high and statistically significant values. For instance, for the currency and commodity in the Australian case both coefficient are very similar($\beta_1^1 = 1.953, \beta_2^2 = 1.931$), whilst in the Canadian case these values are high for the commodity ($\beta_1^1 = 1.698$) with a lower value for the currency ($\beta_2^2 = 0.885$), confirming the importance of the cross-sectional feedback between the intensity of occurrence of extreme events and their magnitudes. Regarding the tail behavior of the distribution functions, the underlying distributions of the marks in all markets exhibit light tail distributions with shape parameters (ξ_1 and ξ_2) greater than zero.

The second competitve specification is the DCC-GARCH-EVT model, where unlike the Hawkes-POT approach, it is necessary to specify both the conditional volatility model and the correlation structure. To this end, a Joint bias test is applied in both markets to determine the existence of asymmetry in the conditional volatility, justifying the use of GJR-GARCH and EGARCH specifications (Engle and Ng, 1993). These results are presented in Table A.2. Table A.4 shows the estimation results for the DCC-GARCH-EVT models. In both countries, a better fit in terms of AIC is obtained in Model 1, which corresponds to a GJR-GARCH specification. As expected, in the autoregressive

part of the mean of the model, the intercept coefficients ϕ^1 and ϕ^2 exhibit low values, while for the autoregressive compomponents, θ^1 and θ^2, their values are higher. In the volatility part of the model, the estimated parameters a^m, b^m, are all positive and indicate persistent volatility. The parameter γ^m capture the asymmetric behavior of the GJR-GARCH model. When this value is positive, the well-known leverage effect occurs, where the response is stronger for a past negative shock than for a positive. For the Canadian case $\gamma^1 = -0.033$ and $\gamma^2 = -0.016$, while for the Australian case $\gamma^1 = -0.059$ and $\gamma^2 = 0.002$. However, in the empirical analysis we work with negative returns, so these markets have the opposite effect to that indicated by the sign of the coefficient, exhibiting therefore a inverse leverage effect in the gold, consistent with Engle (2011) who defines this as a hedge effect. In relation to the parameters of the GDP distribution, the scale parameters β_1 and β_2 are positive and statiscally significant values in both markets, while the shape parameter ξ_1 and ξ_2 are not significantly different from zero. Finally, we analyze the conditional correlations, first we used the Engle-Sheppard CCC χ^2_2 test, to determine the existence of constant or dynamic conditional correlations. These results are displayed in Table A.1. The null hypothesis of conditional correlations is rejected, so exist dynamic conditional correlations between the commodity and the currency in both cases. According to the estimated parameters of the DCC model, the sum of π_1 and π_2 is less than one (Australia:$\pi_1 = 0.044$, $\pi_2 = 0.921$, Canada:$\pi_1 = 0.021$, $\pi_2 = 0.975$), ensuring the conditional covariance matrix H_t to be positive definite. In relation to the chosen multivariate t-Student distribution, in both analyzed markets it is possible at least to establish the existence of the first four moments.

Figure 2 and 3 exhibit the dynamic behavior in-sample of both competitive specifications jointly. The two top panels display the conditional intensity (black line) and the volatility (gray line) for the commodity and currency markets. While the third and fourth panels show the dynamic conditional covariances and correlations, respectively. A first interesting result is that while the conditional intensity uses the occurrence of the extreme events to describe its dynamics, the volatility uses all the information contained in the magnitude of its returns. However, both approaches seem to be complementary to the moment of describing the dynamics of this type of events. In the case of Australia, the intensity of extreme events and their underlying volatility is very similar throughout the period analyzed, with slight growth in gold volatility during the global crisis of 2009. Further, the covariance and conditional correlation exhibit some sudden movements in this same period, showing a strong negative correlation on separate occasions the 2009. Probably due to the occurrence of flight to quality of international investors in financial markets. Particularly in gold market, large losses occur during 2013, when the price reaches around US$1200 per troy ounce (Figuerola-Ferretti and McCrorie, 2016; Białkowski et al., 2015). Consequently, an effect on the Australian dollar can be observed. On the other

Figure 4: VaR estimation to 99% with black line for the Hawkes-POT and gray line for the DCC-GARCH-EVT, applied to the negative log returns of the future gold prices (Top panel) and the Australian dollar (Below panel). The vertical dotted line indicates the division between the in-sample period from 4 January 2005 to 31 December 2014, and the backtesting period from 1 January 2015 until 31 December 2016. The dashed line in the graphic represents the beginning of the backtesting period.

hand, in the case of Canada, the relationship between conditional intensity and volatility follows the same pattern, although also with some differences during the global crisis of 2009, which is more pronounced in the conditional intensity of both markets. An interesting result is also the great jump in the conditional variance during 2009 that does not seem to strongly affect the dynamic conditional correlation, which remains high from 2009 to 2012, showing its lowest value during 2014. According to the classification of Forbes and Rigobon (2002), this significant increase in correlation between Brent market and the Canadian dollar during periods of turmoil would correspond to the definition of contagion, while for the Australian case, this corresponds to interdependence.

3.3. VaR forecasting

Tables A.5-A.6 reports the results of the VaR accuracy tests for the in-sample period from 4 January 2005 to 31 December 2014, and the backtesting period from 1 January 2015 until 31 December 2016. We evaluated the ability of the proposed methodologies to estimate the VaR based on tests LR_{uc}, LR_{ind}, LR_{cc} and DQ_{hit}. The accuracy of the competitive models is evaluated by investigating the dynamic of the VaR exceptions during these periods. Entries in the rows are the significance levels of the respective tests.

In relation to the the in-sample period in the Australian and Canadian cases, slightly better results in VaR accuracy are obtained for the specifacion Hawkes-POT with more than 89% of the p-values of the
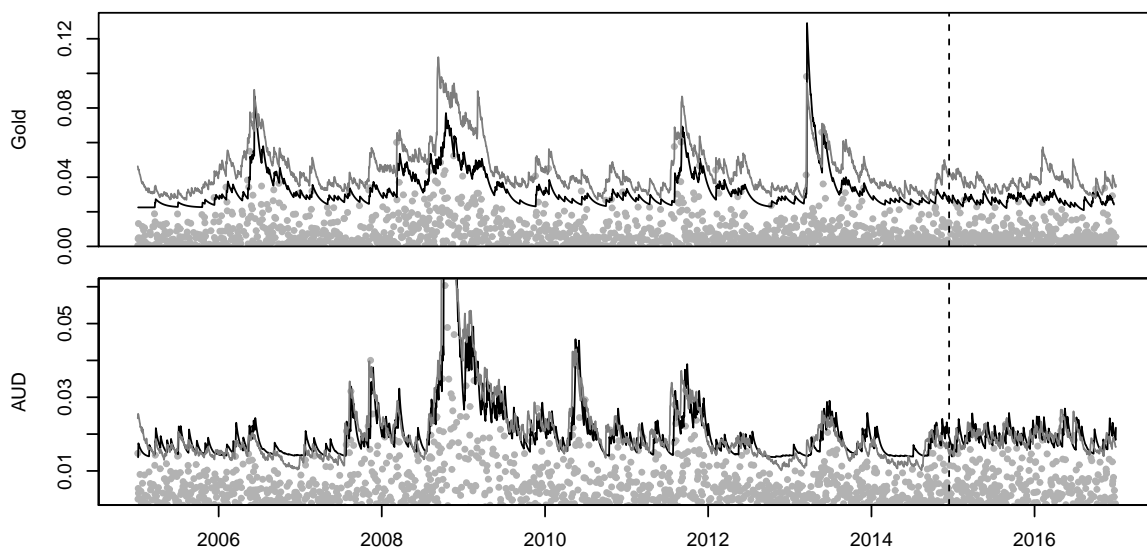
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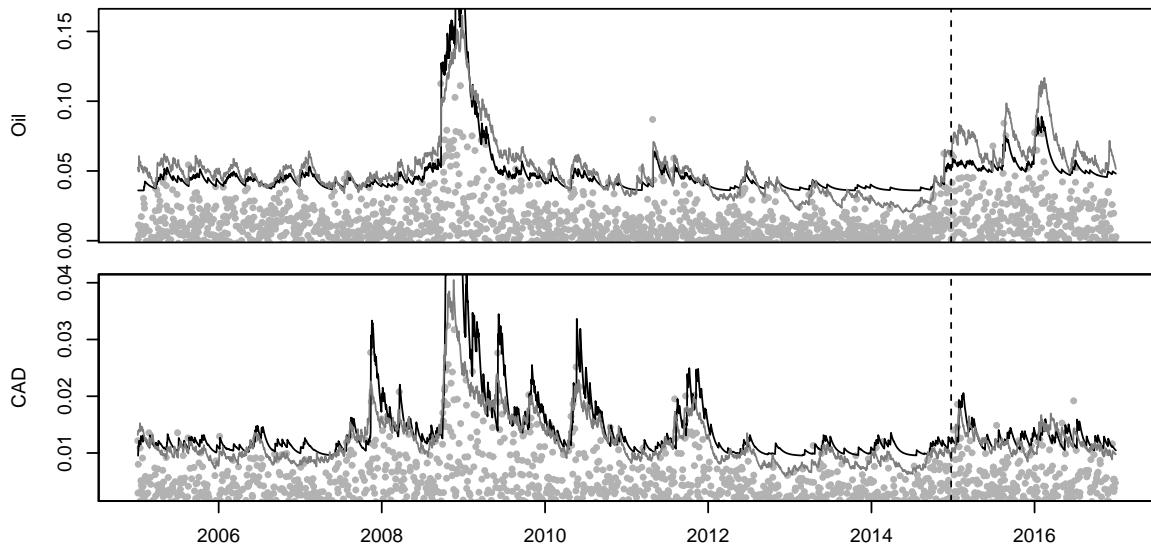
Figure 5: VaR estimation to 99% with black line for the Hawkes-POT and gray line for the DCC-GARCH-EVT, applied to the negative log returns of the future oil prices (Top panel) and the Canadian dollar (Below panel). The vertical dotted line indicates the division between the in-sample period from 4 January 2005 to 31 December 2014, and the backtesting period from 1 January 2015 until 31 December 2016. The dashed line in the graphic represents the beginning of the backtesting period.

different tests no rejecting the null hypothesis. While, in the Backtesting period the results are mixed. In the Australian case, Model 1 in the Hawkes-POT specification obtains excellent results with all test no rejecting the null, while in the Canadian case the Model 1 in the DCC-GARCH-EVT specification allows a better fit with more than 87% of the tests do not reject the null hypothesis. In general, for Australia, the largest complication is the accuracy of the tests of the VaR with low confidence level (0.95) in the commodity and for Canada, the tests are rejected mainly in the currency, with high confidence levels for the VaR.

Figure 4 and 5 display VaR estimates at the 0.99 confidence level with a black and grey line for each chosen model in the Hawkes-POT and DCC-GARCH -EVT specification, respectively. The top panel shows the VaR estimates for the commodity, while the bottom panel for the currency. In the case of Australia for the gold market, we observe that in the in-sample and backtesting period the DCC-GARCH-EVT specification tends to overestimate the VaR regarding the unconditional and conditional coverage test how we noticed in Table A.5. While in the exchange market the dynamic of these VaR estimates are almost identical. For the Canadian market we observe that the behavior of the VaR estimates for the commodity is smoother for the Hawkes-POT specification than the competitive specification, while contrarily, the VaR estimates for the Hawkes-POT approach tends to react more strongly during the crisis of 2009 than the DCC-GARCH-EVT specification.

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4. Conclusion

This paper examines at extreme levels the relationship between two well-known commodities currencies as are the Australian dollar and the Canadian dollar, and the movements in gold and crude oil prices respectively. We consider two perspectives based on extreme value theory, with a multivariate Hawkes-POT marked point process framework for analyzing the intensity and the DCC-GARCH-EVT model to study the conditional correlations following a more traditional approach.

We find that both stochastic processes are complementary. The intensity of the extreme events and the volatility show similar patterns of behavior, throughout the analyzed period, with some changes in its behavior during the crisis of 2009. For the Australian case in this period the volatility increases slightly in the gold market, while for the Canadian case it is possible to observe a considerable increase in the intensity of extreme events in the Oil and the Canadian dollar.

Both models, Hawkes-POT and DCC-GARCH-EVT, collaborate with each other offering participants in these markets a deeper understanding of the links they face this markets in periods of great loss from two different perspectives. With the DCC-GARCH-EVT methodology we find a relationship between the volatility, correlation and dynamics of extreme events, although contemporaneously. While using the Hawkes-POT methodology it is possible to analyze the cross-excitation of extreme events, obtaining a feedback of the relationship between its intensity and magnitude.

An important advantage of the proposed Hawkes-POT model is the flexibility of capturing unidirectional dependence between both markets, where the prices of the main export commodity influence the movements in currency, consistent with previous studies of the literature. Considering the selected model we find that in periods of large losses these currencies are commodity currencies. In both cases, the occurrence rate of extreme events in the commodity has a more relevant role than the magnitude to model the intensity of extreme events in the currency. We also verify that the contribution from cross-excitation on each of the markets is greater than the arrival of exogenous shocks to either the commodity or currency markets in isolation.

A potential avenue for future research is to consider a wider set of representative commodity markets in a country to investigate the different links between them and the channels of contagion during period of crisis. Another line of research could include time varying parameters to capture the behavior of these markets during these turbulent periods of crisis. Possible limitations may be the need for a high number of parameters, which could be restricted in some cases to reduce the curse of dimensionality.

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AppendixA. Tables

		Mean ($\times 1E-03$)	Standard deviation	Min	Max	Skewness	Kurtosis	Jarque-Bera	Box-Pierce	LM-Test
Australia	Currency	0.020	0.009	-0.076	0.082	-0.488	12.569	0.00	0.00	0.00
	Commodity	0.400	0.013	-0.098	0.086	-0.398	5.048	0.00	0.02	0.00
Canada	Currency	0.020	0.006	-0.032	0.039	-0.132	3.212	0.00	0.00	0.00
	Commodity	0.130	0.020	-0.113	0.135	-0.068	3.467	0.00	0.00	0.00

Table A.1: Descriptive statistics of the logarithmic returns of the series and the p-value of the Jarque-Bera and Box-Pierce tests (10 lags).

	Australia		Canada	
	Commodity	Currency	Commodity	Currency
Sign bias test	0.024	0.382	3.107	1.954
	(0.981)	(0.703)	(0.002) ***	(0.051)*
Negative size bias test	1.451	0.933	0.228	1.299
	(0.147)	(0.351)	(0.820)	(0.194)
Positive size bias test	2.299	1.822	0.255	0.709
	(0.022)**	(0.069)*	(0.799)	(0.478)
Joint bias test	14.444	9.492	20.437	8.014
	(0.002)***	(0.023)**	(0.000)***	(0.046)**
Engle-Sheppard CCC χ_2^2 test	64.116		15.392	
	(0.000)***		(0.017)**	

Table A.2: Asymmetry Test and CCC Test. Between parentheses p-value. * , ** , *** indicate the significance at 1, 5 and 10% levels

			Australia					Canada				
			Excep	LR_{uc}	$LRind$	LR_{cc}	DQ_{hit}	Excep	LR_{uc}	$LRind$	LR_{cc}	DQ_{hit}
Hawkes-POT	Model 1	Commodity	0.95	123.00	0.78	0.41	0.69	0.43	135.00	0.56	0.07	0.17	0.08
			0.99	31.00	0.26	0.40	0.37	0.40	25.00	0.89	0.48	0.77	0.49
			0.999	7.00	**0.02**	0.84	0.07	0.84	5.00	0.18	0.89	0.40	0.89
		Currency	0.95	128.00	0.86	0.32	0.60	0.33	126.00	0.82	0.73	0.92	0.74
			0.99	28.00	0.58	0.32	0.52	0.32	29.00	0.52	0.34	0.51	0.34
			0.999	9.00	**0.00**	0.80	**0.01**	0.80	10.00	**0.00**	0.78	**0.00**	0.78
	Model 2	Commodity	0.95	132.00	0.59	0.42	0.62	0.43	129.00	0.96	0.17	0.40	0.19
			0.99	30.00	0.35	0.37	0.43	0.37	23.00	0.59	0.52	0.70	0.52
			0.999	7.00	**0.02**	0.84	0.07	0.84	5.00	0.18	0.89	0.40	0.89
		Currency	0.95	127.00	0.93	0.16	0.36	0.17	126.00	0.82	0.73	0.92	0.74
			0.99	29.00	0.46	0.34	0.49	0.35	29.00	0.52	0.34	0.51	0.34
			0.999	10.00	**0.00**	0.78	**0.00**	0.78	10.00	**0.00**	0.78	**0.00**	0.78
	Model 3	Commodity	0.95	132.00	0.59	0.42	0.62	0.43	129.00	0.96	0.17	0.40	0.19
			0.99	30.00	0.35	0.37	0.43	0.37	23.00	0.59	0.52	0.70	0.52
			0.999	7.00	**0.02**	0.84	0.07	0.84	5.00	0.18	0.89	0.40	0.89
		Currency	0.95	127.00	0.93	0.16	0.36	0.17	126.00	0.82	0.73	0.92	0.74
			0.99	29.00	0.46	0.34	0.49	0.35	28.00	0.65	0.31	0.54	0.31
			0.999	10.00	**0.00**	0.78	**0.00**	0.78	10.00	**0.00**	0.78	**0.00**	0.78
DCC-GARCH-EVT	Model 1	Commodity	0.95	61.00	**0.00**	0.25	**0.00**	0.27	116.00	0.24	0.44	0.38	0.45
			0.99	9.00	**0.00**	**0.02**	**0.00**	**0.02**	21.00	0.33	0.56	0.53	0.56
			0.999	1.00	0.27	0.98	0.55	0.98	2.00	0.71	0.96	0.93	0.96
		Currency	0.95	114.00	0.25	0.41	0.37	0.42	180.00	**0.00**	0.64	**0.00**	0.63
			0.99	29.00	0.47	0.34	0.49	0.35	66.00	**0.00**	0.55	**0.00**	0.56
			0.999	6.00	0.06	0.87	0.18	0.87	13.00	**0.00**	0.72	**0.00**	0.72
	Model 2	Commodity	0.95	61.00	**0.00**	0.25	**0.00**	0.27	126.00	0.81	0.46	0.74	0.47
			0.99	9.00	**0.00**	**0.02**	**0.00**	**0.02**	22.00	0.45	0.54	0.62	0.54
			0.999	1.00	0.27	0.98	0.55	0.98	2.00	0.71	0.96	0.93	0.96
		Currency	0.95	108.00	0.08	0.28	0.12	0.29	166.00	**0.00**	0.58	**0.00**	0.58
			0.99	26.00	0.89	0.27	0.54	0.27	63.00	**0.00**	0.63	**0.00**	0.63
			0.999	7.00	**0.02**	**0.01**	**0.00**	**0.01**	12.00	**0.00**	0.74	**0.00**	0.74
	Model 3	Commodity	0.95	61.00	**0.00**	0.25	**0.00**	0.27	111.00	0.10	0.16	0.10	0.17
			0.99	9.00	**0.00**	**0.02**	**0.00**	**0.02**	24.00	0.73	0.50	0.75	0.50
			0.999	1.00	0.27	0.98	0.55	0.98	3.00	0.80	0.93	0.96	0.93
		Currency	0.95	116.00	0.33	0.46	0.48	0.48	177.00	**0.00**	0.50	**0.00**	0.50
			0.99	27.00	0.74	0.29	0.54	0.30	71.00	**0.00**	0.44	**0.00**	0.44
			0.999	8.00	**0.01**	0.82	**0.02**	0.82	15.00	**0.00**	0.67	**0.00**	0.68

Table A.5: VaR accuracy tests for the Hawkes-POT and DCC-GARCH-EVT models for the in-sample period. Entries in the rows are the significance levels (p-values) of the respective tests, with the exception of the confidence level α for the VaR and the number of exceptions (Excep).

				Australia					Canada				
				Excep	LR_{uc}	$LRind$	LR_{cc}	DQ_{hit}	Excep	LR_{uc}	$LRind$	LR_{cc}	DQ_{hit}
Hawkes-POT	Model 1	Commodity	0.95	31	0.27	0.05	0.08	0.05	34	0.10	0.33	0.16	0.33
			0.99	7	0.42	0.68	0.66	0.68	10	0.05	0.55	0.13	0.53
			0.999	1	0.54	0.95	0.83	0.95	4	**0.00**	0.83	**0.01**	0.80
		Currency	0.95	31	0.27	0.48	0.42	0.46	36	**0.04**	0.73	0.12	0.74
			0.99	6	0.69	0.05	0.14	0.05	10	0.05	**0.01**	**0.01**	**0.01**
			0.999	0	0.31	1.00	0.60	1.00	3	**0.02**	0.85	0.06	0.85
	Model 2	Commodity	0.95	33	0.14	**0.03**	**0.04**	**0.04**	34	0.10	0.33	0.16	0.33
			0.99	7	0.42	0.68	0.66	0.68	9	0.11	0.59	0.25	0.57
			0.999	1	0.54	0.95	0.83	0.95	4	**0.00**	0.83	**0.01**	0.80
		Currency	0.95	31	0.27	0.48	0.42	0.46	36	**0.04**	0.73	0.12	0.74
			0.99	6	0.69	0.05	0.14	0.05	10	0.05	**0.01**	**0.01**	**0.01**
			0.999	0	0.31	1.00	0.60	1.00	3	**0.02**	0.85	0.06	0.85
	Model 3	Commodity	0.95	33	0.14	**0.03**	**0.04**	**0.04**	34	0.10	0.33	0.16	0.33
			0.99	7	0.42	0.68	0.66	0.68	9	0.11	0.59	0.25	0.57
			0.999	1	0.54	0.95	0.83	0.95	4	**0.00**	0.83	**0.01**	0.80
		Currency	0.95	31	0.27	0.48	0.42	0.46	36	**0.04**	0.73	0.12	0.74
			0.99	6	0.69	0.05	0.14	0.05	10	0.05	**0.01**	**0.01**	**0.01**
			0.999	0	0.31	1.00	0.60	1.00	3	**0.02**	0.85	0.06	0.85
DCC-GARCH-EVT	Model 1	Commodity	0.95	9	**0.00**	0.57	**0.00**	0.58	24	0.68	0.13	0.29	0.14
			0.99	2	0.11	0.90	0.27	0.90	3	0.29	0.85	0.56	0.85
			0.999	0	0.31	1.00	0.59	1.00	0	0.31	1.00	0.59	1.00
		Currency	0.95	28	0.70	0.64	0.83	0.65	37	**0.04**	0.66	0.11	0.67
			0.99	5	0.93	0.76	0.95	0.76	11	**0.03**	0.22	**0.04**	0.22
			0.999	1	0.56	0.95	0.84	0.95	2	0.12	0.90	0.29	0.90
	Model 2	Commodity	0.95	9	**0.00**	0.57	**0.00**	0.58	21	0.29	0.18	0.24	0.20
			0.99	2	0.11	0.90	0.27	0.90	3	0.29	0.85	0.56	0.85
			0.999	0	0.31	1.00	0.59	1.00	0	0.31	1.00	0.59	1.00
		Currency	0.95	27	0.85	0.71	0.92	0.72	32	0.25	0.98	0.51	0.98
			0.99	4	0.58	0.80	0.83	0.80	10	0.06	0.18	0.07	0.18
			0.999	1	0.56	0.95	0.84	0.95	4	**0.00**	**0.02**	**0.00**	**0.02**
	Model 3	Commodity	0.95	9	**0.00**	0.57	**0.00**	0.58	23	0.53	0.14	0.28	0.16
			0.99	2	0.11	0.90	0.27	0.90	5	0.93	0.76	0.95	0.76
			0.999	0	0.31	1.00	0.59	1.00	0	0.31	1.00	0.59	1.00
		Currency	0.95	28	0.70	0.64	0.83	0.65	39	**0.01**	0.96	0.05	0.96
			0.99	4	0.58	0.80	0.83	0.80	12	**0.01**	0.27	**0.02**	0.27
			0.999	1	0.56	0.95	0.84	0.95	4	**0.00**	**0.02**	**0.00**	**0.02**

Table A.6: VaR accuracy tests for the Hawkes-POT and DCC-GARCH-EVT models for the out-sample period. Entries in the rows are the significance levels (p-values) of the respective tests, with the exception of the confidence level α for the VaR and the number of exceptions (Excep).

AppendixB. Accuracy of the VaR

AppendixB.1. Likelihood ratio unconditional coverage test (LR_{uc})

The aim of the LR_{uc} test is to determine whether the number of errors obtained differs from the expected value, remaining consistent with the stated confidence level α in the risk quantification. Consider $H_t = I(r_t < -VaR^t_\alpha)$ as being the sequence of efficient predictions of the *VaR*; if $E[H_t] = \alpha$ with $H_t \overset{iid}{\sim} Bernoulli(\alpha)$, then under the null hypothesis $H_0 : E[H_t] = \alpha$, for which the likelihood is defined as

$$\ln L(\alpha; H_1, H_2, \ldots, H_T) = (1-\alpha)^{n_0} \alpha^{n_1}$$

where n_0 is the number of correct predictions, while n_1 is the number of violations of *VaR*. On the other hand, the alternative hypothesis is defined as $H_1 : E[H_t] \neq \alpha$,

$$\ln L(\hat{\pi}; H_1, H_2, \ldots, H_T) = (1-\pi)^{n_0} \pi^{n_1}$$

where $\hat{\pi} = \frac{n_1}{n_0+n_1}$ is the likelihood of π. Then, by means of a likelihood ratio test, one can test the unconditional coverage

$$LR_{uc} = 2 \left[\ln L(\hat{\pi}; H_1, H_2, \ldots, H_T) - \ln L(\alpha; H_1, H_2, \ldots, H_T) \right] \overset{asy}{\sim} \chi^2_1$$

with L being the likelihood of the binomial distribution and χ^2_1 the Chi-squared distribution with one degree of freedom.

AppendixB.2. Likelihood ratio test of independence (LR_{ind})

Repeated mistakes result in significant losses for the investor, the LR_{ind} test verifies that there is no dependence over time between violations. Since H_t is a series of binary variables, we can model the dependence as a Markov chain whose first order transition matrix is defined by

$$\Pi = \begin{bmatrix} 1 - \pi_{01} & \pi_{01} \\ 1 - \pi_{11} & \pi_{11} \end{bmatrix}$$

where $n_{ij} = \sum I(H_t = j | H_{t-1} = i)$ represents the number of transitions of state i to state j. Under the null hypothesis, we observe that $\pi_{01} = \pi_{11} = \pi_0$, so the conditional likelihood at the first state has to be

$$L(\Pi_1; H_2, \ldots, H_T | H_1) = (1 - \pi_{01})^{n_{00}+n_{10}} \pi_{01}^{n_{01}+n_{11}}.$$

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Under the alternative hypothesis, we have that $\hat{\pi}_{01} = \frac{n_{00}}{n_{01}+n_{01}}$ and $\hat{\pi}_{11} = \frac{n_{11}}{n_{10}+n_{11}}$ with likelihood

$$L(\Pi_2; H_2, \ldots, H_T|H_1) = (1-\pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1-\pi_{11})^{n_{10}} \pi_{11}^{n_{11}}$$

Finally, the likelihood ratio test of independence for this statistic is defined by

$$LR_{ind} = 2[\ln L(\Pi_2; H_2, \ldots, H_T|H_1) - \ln L(\Pi_1; H_2, \ldots, H_T|H_1)] \overset{asy}{\sim} \chi_1^2$$

which is asymptotically Chi-square distributed with one degree of freedom.

AppendixB.3. Likelihood ratio test of conditional coverage(LR_{cc})

Providing a more global perspective, the LR_{cc} simultaneously checks the previous two tests

$$LR_{cc} = LR_{uc} + LR_{ind}$$

where the likelihood ratio test is defined as

$$LR_{cc} = 2[\ln L(\Pi_2; H_2, \ldots, H_T|H_1) - \ln L(\alpha; H_2, \ldots, H_T|H_1)] \overset{asy}{\sim} \chi_2^2$$

which is asymptotically Chi-squared distributed with two degrees of freedom. For further details regarding these three tests, see Christoffersen (1998).

AppendixB.4. Dynamic quantile test (DQ_{hit})

Evaluate the existence of a correlation between VaR violations based on logit models. The number of violations of the VaR with mean equal to 0 we defined as the Hit in time t, (i.e.,$Hit_t = H_t - \alpha$ with $E[Hit_t] = 0$). Then, the model to be estimated is as follows:

$$Hit_{t+1} = a + bHit_t + e_t,$$

where e_t are discrete random iid. Under the null hypothesis $H_0 = a = b = 0$, regressors should not have explanatory power and are asymptotically Chi-squared distributed with one degree of freedom.

Table A.3 — Generalized Pareto Distribution Function / Ground Process

| | | | Ground Process | | | | | | | | | | | | Generalized Pareto Distribution Function | | | | | | | | | |
| | | Commodity | | | | | | | | Currency | | | | Commodity | | | | Currency | | | | | |
		μ_1	η_{11}	η_{12}	α_1	δ_{11}	δ_{12}	δ_{22}	δ_{21}	μ_2	η_{22}	η_{21}	α_2	β_0^1	β_1^1	β_2^1	ξ_1	β_0^2	β_2^2	β_1^2	ξ_2	Log-Lik	AIC
Australia	1	0.060	0.118	0.083	0.061	0.487	0.493	0.380	0.001	0.046	0.315	0.071	0.072	0.617	1.523	0.001	0.123	0.320	1.993	0.001	0.209	-1950.822	3941.644
		(0.014)	(0.116)	(0.081)	(0.042)	(0.091)	(0.170)	(0.053)	(0.003)	(0.010)	(0.098)	(0.013)	(0.021)	(0.110)	(0.577)	(0.001)	(0.062)	(0.058)	(0.339)	(0.001)	(0.069)		
	2	0.050	0.283		0.041	0.416		0.389	0.001	0.044	0.284	0.128	0.080	0.561	1.953		0.094	0.332	1.931	0.001	0.221	-1954.675	3943.349
		(0.010)	(0.074)		(0.012)	(0.050)		(0.054)	(0.002)	(0.010)	(0.091)	(0.013)	(0.022)	(0.091)	(0.442)		(0.063)	(0.057)	(0.324)	(0.001)	(0.068)		
	3	0.050	0.283		0.041	0.416		0.389	0.389	0.044	0.284	0.128	0.080	0.561	1.953		0.094	0.332	1.931		0.221	-1954.675	3939.350
		(0.010)	(0.074)		(0.012)	(0.050)		(0.054)	(0.054)	(0.010)	(0.091)	(0.013)	(0.022)	(0.091)	(0.442)		(0.063)	(0.057)	(0.324)		(0.068)		
Canada	1	0.057	0.136	0.066	0.046	0.406	0.750	1.305	0.077	0.049	0.157	0.133	0.055	0.870	1.564	0.001	0.156	0.178	0.881	0.001	0.186	-1940.332	3920.664
		(0.011)	(0.071)	(0.107)	(0.013)	(0.069)	(1.422)	(0.183)	(0.460)	(0.011)	(0.063)	(0.015)	(0.013)	(0.116)	(0.588)	(0.001)	(0.044)	(0.033)	(0.230)	(0.001)	(0.046)		
	2	0.060	0.174		0.044	0.389	0.074	1.304		0.049	0.157	0.136	0.054	0.853	1.698		0.150	0.178	0.883	0.001	0.187	-1940.927	3915.854
		(0.010)	(0.071)		(0.011)	(0.064)	(0.452)	(0.183)		(0.011)	(0.063)	(0.015)	(0.013)	(0.114)	(0.587)		(0.044)	(0.033)	(0.231)	(0.001)	(0.046)		
	3	0.060	0.174		0.044	0.389		1.303		0.048	0.160	0.151	0.054	0.853	1.698		0.150	0.177	0.885		0.187	-1940.984	3911.968
		(0.010)	(0.071)		(0.011)	(0.064)		(0.181)		(0.010)	(0.060)	(0.012)	(0.012)	(0.114)	(0.587)		(0.044)	(0.033)	(0.230)		(0.046)		

Table A.3: Results of the Hawkes-POT models for Australia and Canada for the in-sample period. Negative logarithmic returns are applied to future prices of gold (subscript 1) and the Australian dollar (subscript 2), and Oil (subscript 2), and the Canadian dollar (subscript 2), respectively. Standard errors are in parentheses. Log-lik corresponds to the log-likelihood, while AIC corresponds to the value of the Akaike information criterion.

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Table A.4: Results of the DCC-GARCH-EVT models for Australia and Canada in the period In-Sample.

	AR(1) - GARCH(1,1) Commodity						AR(1) - GARCH(1,1) Currency						DCC Joint			EVT Commodity		EVT Currency			
	ϕ^1	θ^1	ω^1	a^1	b^1	γ^1	ϕ^2	θ^2	ω^2	a^2	b^2	γ^2	π_1	π_2	ν	β_1	ξ_1	β_2	ξ_2	Log-Lik	AIC
Australia 1	-4.E-04 (2.E-04)	0.003 (0.026)	2.E-06 (9.E-06)	0.051 (0.085)	0.938 (0.088)	0.002 (0.021)	6.E-05 (1.E-04)	-0.040 (0.021)	1.E-06 (0.014)	0.086 (0.014)	0.934 (0.017)	-0.059 (0.025)	0.044 (0.011)	0.921 (0.026)	7.023 (0.797)	0.627 (0.058)	0.101 (0.067)	0.671 (0.057)	-0.076 (0.057)	16574.356	-33116.712
Australia 2	-5.E-04 (3.E-04)	0.004 (0.025)	-0.134 (0.007)	0.008 (0.003)	0.984 (0.000)	0.128 (0.031)	8.E-05 (1.E-04)	-0.049 (0.022)	-0.086 (0.012)	0.045 (0.011)	0.991 (0.001)	0.143 (0.025)	0.043 (0.010)	0.926 (0.022)	7.042 (0.542)	0.615 (0.056)	0.103 (0.066)	0.664 (0.057)	-0.064 (0.057)	16571.499	-33111.000
Australia 3	-4.E-04 (3.E-04)	0.003	2.E-06 (8.E-06)	0.052 (0.072)	0.938 (0.080)		-5.E-05 (1.E-04)	-0.043 (0.022)	1.E-06 (5.E-06)	0.073 (0.011)	0.920 (0.001)		0.044 (0.010)	0.921 (0.022)	6.894 (0.707)	0.624 (0.056)	0.104 (0.066)	0.697 (0.057)	-0.102 (0.059)	16561.672	-33095.345
Canada 1	-3.E-05 (3.E-04)	-0.041 (0.020)	1.E-06 (1.E-06)	0.059 (0.008)	0.956 (0.004)	-0.033 (0.010)	1.E-05 (1.E-04)	-0.030 (0.020)	1.E-06 (1.E-06)	0.060 (0.016)	0.941 (0.015)	-0.016 (0.013)	0.021 (0.006)	0.975 (0.008)	13.142 (1.941)	0.555 (0.047)	0.006 (0.058)	0.649 (0.052)	-0.162 (0.054)	16628.098	-33224.197
Canada 2	-1.E-05 (3.E-04)	-0.033 (0.020)	-0.041 (0.002)	0.035 (0.007)	0.995 (0.000)	0.089 (0.005)	3.E-05 (9.E-05)	-0.032 (0.019)	-0.082 (0.015)	0.015 (0.012)	0.992 (0.001)	0.120 (0.032)	0.021 (0.006)	0.976 (0.008)	13.066 (1.876)	0.564 (0.047)	-0.009 (0.058)	0.671 (0.052)	-0.172 (0.053)	16618.021	-33204.043
Canada 3	-2.E-04 (3.E-04)	-0.042 (0.021)	1.E-06 (2.E-06)	0.045 (0.014)	0.952 (0.014)		-9.E-06 (9.E-05)	-0.030 (0.020)	1.E-06 (1.E-06)	0.053 (0.015)	0.940 (0.016)		0.019 (0.006)	0.978 (0.008)	13.017 (1.955)	0.593 (0.049)	-0.024 (0.056)	0.659 (0.055)	-0.174 (0.054)	16612.391	-33196.781

Table A.4: Results of the DCC-GARCH-EVT models for Australia and Canada in the period In-Sample. Negative logarithmic returns are applied to future prices of gold (subscript 1) and the Australian dollar (subscript 2), and Oil (subscript 1) and the Canadian dollar (subscript 2), respectively. Standard errors are in parentheses. Log-lik corresponds to the log-likelihood, while AIC corresponds to the value of the Akaike information criterion.