Point process models for extreme returns: Harnessing implied volatility

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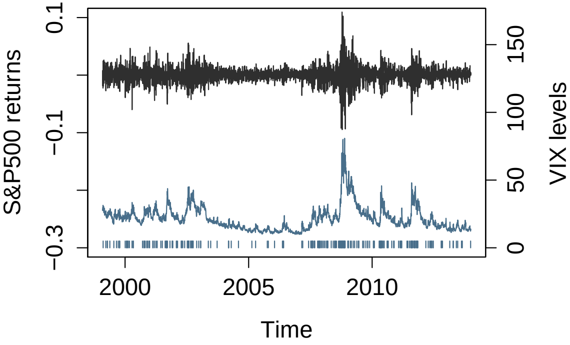
Introduction

- Clustering of extreme events
- Reliable risk measures[BCBS, 2012]

Let the tail speak for itself!

Motivation

- **The CBOE Volatility Index® (IV)** is a key measure of market expectations of near-term volatility conveyed by S&P 500 stock index option prices (The VIX).
- Since its introduction in 1993, IV has been considered by many to be the world's premier barometer of **investor sentiment and market volatility**.



Motivation

- **The CBOE Volatility Index® (IV)** is a key measure of market expectations of near-term volatility conveyed by S&P 500 stock index option prices (The VIX).
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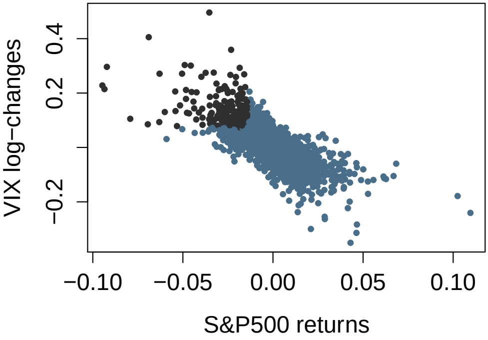

Financial extreme events features

Summary:

- Extremes appear in clusters
- Excess over a high threshold highly correlated
- Inter-exceedance times are correlated
- Relationship between size of the exceedances and last elapsed inter-exceedance times
- IV indices are negatively correlated with stock market indices

Contribution

Research Questions :

1. How do extreme shocks in an IV index relate to extreme events in its respective stock market return?

1. How can the occurrence and intensity of extreme events in IV indices influence the dynamic behavior on stock market returns and vice versa?

Contribution

Approach : Utilise IV within intensity based point process models for extreme returns.

- Model 1: IV as an exogenous variable influencing the intensity and the size distribution of extreme events.
- Model 2. Extreme movements in IV are treated as events themselves, with their impact on extreme events in equity returns captured through a bivariate Hawkes model.
- $\Longrightarrow$Forecasting extreme losses within a Value-at-Risk framework.
- The benchmark $\Longrightarrow$IV within the GARCH-EVT framework.

Features

- Temporal clustering of both the occurrence of extremes and the size thereof
- Cross-sectional feedback between individual exceedance intensities and
- Feedback between the magnitude of exceedances and their intensity.

Date Description

- The data consists of daily returns for the S&P 500, Nasdaq, DAX 30, Dow Jones and Nikkei stock market indices, and their respective IV indices. Aall series ending December 31, 2013

Outline

Outline

Literature Review

IV indices are an important measure of short-term expected risk

- Blair et al. [2001], Poon and Granger [2003] IV as an exogenous variable in GARCH models $\Longrightarrow$ beneficial in terms of forecasting.
- Becker et al. [2009] IV contain useful information about future jump activity in returns $\Longrightarrow$reflect extreme movements in prices.
- Hilal et al. [2011] conditional approach for extremal dependence between daily returns on VIX futures and S&P500$\Longrightarrow$ VIX futures returns are very sensitive to stock market downside risk.
- Peng and Ng [2012] cross-market dependence of five of the most important equity markets and their corresponding volatility indices $\Longrightarrow$ existence of an asymmetric tail dependence.
- Aboura and Wagner [2014] dependence between S&P 500 index returns and VIX index changes $\Longrightarrow$existence of a contemporaneous volatility-return tail dependence for (-) extreme events though not for (+) returns.

Outline

Univariate Hawkes-POT model

Basic setting

- Let $\{(X_t, Y_t)\}_{t\geq 1}$ be a vector of r.v that represent the log-returns of a stock market index and the associated IV index.
- Define a finite subset of observations $\{(T_i, W_i, Z_i)\}_{i\geq 1}$, where:

$\Longrightarrow T_i$ corresponds to occurrence times
$\Longrightarrow W_i$ magnitude of exceedances higher then $u > 0$ ($W_i := X_{T_i} - u$)
$\Longrightarrow Z_i$ the covariate obtained from the IV index ($Z_i := Y_{T_i}$)

- A Marked Point Process (MPP) $N(t) := N(0,t] = \sum_{i\geq 1} \mathbb{1}\{T_i \leq t, W_i = w\}$ with past history or natural filtration $\mathscr{H}_t = \{(T_i, W_i, Z_i)\ \forall i : T_i < t\}$

$$\lambda(t, w \mid \mathscr{H}_t) = \lambda_g(t \mid \mathscr{H}_t)\, g(w \mid \mathscr{H}_t, t), \tag{1}$$

Univariate Hawkes-POT model

- The conditional intensity is characterized by the branching structure of a Hawkes process
$$\lambda_g\left(t \mid \mathscr{H}_t\right) = \mu + \eta \sum_{i:t_i<t} e^{\delta w_i + \rho z_i} \gamma e^{-\gamma(t-t_i)}, \tag{2}$$
- Motivated by the Pickands–Balkema–de Haan's theorem, the extreme losses are assumed to follow a conditional Generalized Pareto Distribution (GPD)

$$g\left(w \mid \mathscr{H}_t, t\right) = \begin{cases} \frac{1}{\beta(w|\mathscr{H}_t,t)}\left(1+\xi\frac{w}{\beta(w|\mathscr{H}_t,t)}\right)^{-1/\xi-1} & , \quad \xi \neq 0 \\ \frac{1}{\beta(w|\mathscr{H}_t,t)}\exp\left(-\frac{w}{\beta(w|\mathscr{H}_t,t)}\right) & , \quad \xi = 0, \end{cases} \quad , \tag{3}$$

where ξ is the shape parameter and $\beta\left(w \mid \mathscr{H}_t, t\right)$ is a scale parameter specified as a self-exciting function

$$\beta\left(w \mid \mathscr{H}_t, t\right) = \beta_0 + \beta_1 \sum_{i:t_i<t} e^{\delta w_i + \rho z_i} \gamma e^{-\gamma(t-t_i)}.$$

Bivariate Hawkes-POT model

Basic setting

- Let $\{(X_t, Y_t)\}_{t\geq 1}$ be a vector of r.v that represent the log-returns of a stock market index and the associated IV index.
- MPP is defined as a vector of point processes $\mathbf{N}(t) : \{N_1(t), N_2(t)\}$

$\Longrightarrow N_1(t)$ is defined through the pairs$\left\{\left(T_i^1, W_i\right)\right\}_{i\geq 1}$; the subset of extreme events in the log-returns of the stock market occurring at time T_i^1 over a high threshold $u_1 > 0$, with $W_i := X_{T_i^1} - u_1$.
$\Longrightarrow N_2(t)$ is defined by the pairs of events $\left\{\left(T_i^2, Z_i\right)\right\}_{i\geq 1}$with $Z_i := Y_{T_i^2} - u_2$, which also characterizes the subset of extreme events occurring in IV at time T_i^2 over a high threshold $u_2 > 0$.

- $\mathscr{H}_t = \left\{\left(T_i^1, W_i\right), \left(T_j^2, Z_j\right) \ \forall i, j : T_i^1 < t \wedge T_j^2 < t\right\}$ denotes the combined history over all times and marks

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Bivariate Hawkes-POT model

- This bivariate MPP includes a bivariate ground process $N_k^g(t) := \sum_{i\geq 1} \mathbb{1}\left\{T_i^k \leq t\right\}$ with conditional intensity

$$\begin{aligned} \lambda_g^1(t \mid \mathscr{H}_t) &= \mu_1 + \eta_{11} \sum_{i:t_i^1<t} e^{\delta w_i} \gamma_1 e^{-\gamma_1\left(t-t_i^1\right)} + \eta_{12} \sum_{i:t_i^2<t} e^{\rho z_i} \gamma_2 e^{-\gamma_2\left(t-t_i^2\right)} \quad (4) \\ \lambda_g^2(t \mid \mathscr{H}_t) &= \mu_2 + \eta_{21} \sum_{i:t_i^1<t} e^{\delta w_i} \gamma_1 e^{-\gamma_1\left(t-t_i^1\right)} + \eta_{22} \sum_{i:t_i^2<t} e^{\rho z_i} \gamma_2 e^{-\gamma_2\left(t-t_i^2\right)} \end{aligned}$$

- Similar to the univariate MPP we also consider a generalized Pareto density for the stock market returns as in (3), but with conditional scale parameter

$$\beta(w \mid \mathscr{H}_t, t) = \beta_0 + \beta_1 \sum_{i:t_i^1<t} e^{\delta w_i} \gamma_1 e^{-\gamma_1\left(t-t_i^1\right)} + \beta_{12} \sum_{i:t_i^2<t} e^{\rho z_i} \gamma_2 e^{-\gamma_2\left(t-t_i^2\right)}. \quad (5)$$

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Conditional mean and volatility models

The conditional mean of the equity market returns is specified as an Auto Regressive Moving Average (ARMA) process

$$r_t = \mu + \sum_{i=1}^{m} a_i r_{t-i} + \sum_{j=1}^{n} b_j \varepsilon_{t-j} + \varepsilon_t. \tag{6}$$

Where r_t denotes the return on a stock market index at time t, μ the mean, a_i and b_j describe the autoregressive and moving average coefficients, respectively and ε_t denotes the residual term. The residuals are defined by

$$\varepsilon_t = \eta_t \sqrt{h_t}, \qquad \eta_t \sim iid(0,1), \tag{7}$$

where η_t is the standardized residual and h_t is the conditional variance.

Conditional mean and volatility models

The GARCH specifications considered for the conditional variances which include IV as an exogenous variable are

$$\begin{aligned} \text{GARCH(1,1)}: & & \quad (8)\\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} + \gamma IV_{t-1} \\ \text{GJR-GARCH(1,1)}: & & \quad (9)\\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \delta \max\left(0, -\varepsilon_{t-1}\right)^2 + \beta h_{t-1} + \gamma IV_{t-1} \\ \text{EGARCH(1,1)}: & & \quad (10)\\ \ln h_t &= \omega + \alpha \varepsilon_{t-1} + \delta\left(\left|\varepsilon_{t-1}\right| - E\left|\varepsilon_{t-1}\right|\right) + \beta \ln h_{t-1} + \gamma \ln IV_{t-1}. \end{aligned}$$

These three conditional volatility specifications are estimated under the following two alternative distributions, namely Student-t and Skew Student-t.

Outline

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Conditional risk measures

GARCH-EVT Models

- The corresponding VaR at the α confidence level of the assumed distribution of the residuals η_t , i.e., $VaR_{\alpha}(\eta_t) : \inf\{x \in \mathbb{R} : P(\eta_t > x) \leq 1-\alpha\}$

$$VaR_{\alpha}^{t} = \mu_{t-1} + VaR_{\alpha}(\eta)\sigma_{t-1},$$

where $\mu_{t-1} = \mu + \sum_{i=1}^{m} a_i r_{t-i}$ and $\sigma_{t-1} = \sum_{j=1}^{n} b_j \sqrt{h_{t-j}} + \sqrt{h_t}$.

Hawkes-POT Models

- A prediction of the VaR in the next instant at the α confidence level is given by

$$\mathrm{VaR}_{\alpha}^{t_{i+1}} = u + \frac{\beta\left(w \mid \mathscr{H}_t\right)}{\xi}\left\{\left(\frac{\lambda_g(t_{i+1} \mid \mathscr{H}_t)}{1-\alpha}\right)^{\xi} - 1\right\}.$$

Outline

Empirical results

Data	Model	μ	η	α	δ	ρ	β_0	β_1	ξ	Log. like	AIC
S&P500 -VIX	1	0.031	0.386	0.046	27.660	5.351	0.004	0.017	-0.097	446.904	-877.807
		(0.005)	(0.049)	(0.008)	(3.408)	(2.006)	(0.001)	(0.002)	(0.034)		
	2	0.033	-0.449	0.054	32.389		0.004	0.019	-0.092	443.718	-873.435
		(0.005)	(0.050)	(0.008)	(2.944)		(0.001)	(0.002)	(0.035)		
	3	0.021	0.794	0.038			0.004	0.030	0.043	410.805	-809.609
		(0.004)	(0.054)	(0.006)			(0.001)	(0.003)	(0.038)		
DAX - VDAX	1	0.044	0.259	0.073	43.010	1.655	0.005	0.016	-0.179	333.364	-650.727
		(0.005)	(0.046)	(0.010)	(4.257)	(1.026)	(0.001)	(0.003)	(0.033)		
	2	0.044	0.294	0.074	43.319		0.005	0.018	-0.180	332.085	-650.170
		(0.005)	(0.045)	(0.010)	(4.076)		(0.001)	(0.003)	(0.034)		
	3	0.027	0.736	0.050			0.004	0.042	-0.034	292.604	-573.208
		(0.005)	(0.057)	(0.007)			(0.001)	(0.004)	(0.035)		
Nikkei - VXJ	1	0.060	0.164	0.066	22.564	2.927	0.005	0.023	-0.101	150.623	-285.246
		(0.009)	(0.067)	(0.012)	(5.068)	(0.906)	(0.001)	(0.004)	(0.050)		
	2	0.060	0.207	0.078	31.455		0.004	0.025	-0.029	146.767	-279.534
		(0.009)	(0.072)	(0.016)	(4.118)		(0.001)	(0.004)	(0.040)		
	3	0.034	0.630	0.047			0.004	0.038	0.133	121.026	-230.052
		(0.008)	(0.093)	(0.012)			(0.001)	(0.007)	(0.059)		

Table: Estimates of the univariate Hawkes-POT models used for the analysis of the cluster behavior for extreme events of negative log-returns in stock markets and positive level-changes in IV indices. Standard errors are in parenthesis. Log.like corresponds to the log-likelihood of the model. AIC is the Akaike Information Criterion.

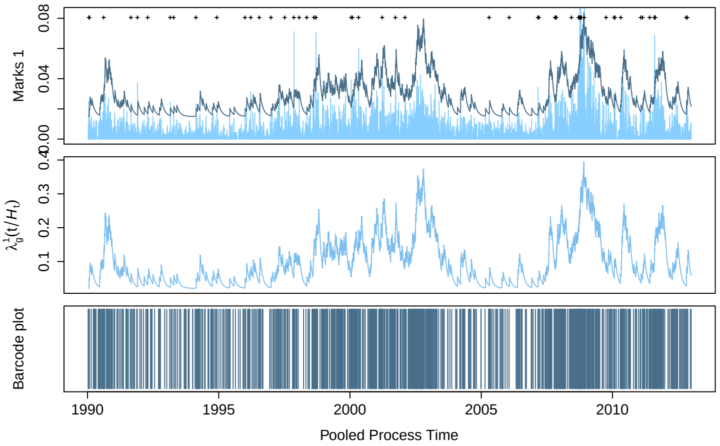
Empirical results

Data	Model	μ_1	η_{11}	η_{12}	α_1	δ	μ_2	η_{21}	η_{22}	α_2	ρ	β_0	β_1	β_{12}	ξ	log. like	AIC
S&P500 - VIX	1	0.034	0.445	0.000	0.054	32.647	0.065	0.025	0.318	0.031	0.001	0.004	0.019	0.001	-0.093	-1461.07	2950.14
		(0.005)	(0.050)	(0.003)	(0.008)	(2.933)	(0.010)	(0.017)	(0.118)	(0.014)	(0.002)	(0.001)	(0.002)	(0.001)	(0.035)		
	2	0.034	0.445	0.000	0.054	32.647	0.065	0.025	0.318	0.031		0.004	0.019		-0.093	-1461.07	2946.14
		(0.005)	(0.050)	(0.003)	(0.008)	(2.933)	(0.010)	(0.017)	(0.118)	(0.014)		(0.000)	(0.002)		(0.035)		
	3	0.020	0.803	0.000	0.035		0.063	0.000	0.378	0.029		0.003	0.023	0.019	0.035	-1491.98	3007.96
		(0.004)	(0.055)	(0.005)	(0.006)		(0.010)	(0.002)	(0.099)	(0.010)		(0.001)	(0.005)	(0.009)	(0.037)		
DAX - VDAX	1	0.000	0.180	0.643	0.094	49.785	0.041	0.023	0.577	0.004	0.002	0.006	0.015	0.001	-0.192	-1357.04	2742.08
		(0.007)	(0.039)	(0.070)	(0.013)	(4.593)	(0.010)	(0.014)	(0.093)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.034)		
	2	0.000	0.180	0.644	0.094	49.785	0.041	0.023	0.577	0.004		0.006	0.015		-0.192	-1357.04	2738.08
		(0.007)	(0.040)	(0.110)	(0.013)	(4.604)	(0.011)	(0.015)	(0.126)	(0.001)		(0.001)	(0.002)		(0.034)		
	3	0.017	0.699	0.137	0.042		0.051	0.001	0.493	0.039		0.003	0.037	0.017	-0.055	-1476.20	2976.40
		(0.007)	(0.068)	(0.089)	(0.009)		(0.010)	(0.061)	(0.116)	(0.014)		(0.001)	(0.005)	(0.008)	(0.039)		
Nikkei - VXJ	1	0.057	0.168	0.068	0.080	33.449	0.066	0.019	0.318	0.066	0.003	0.005	0.023	0.001	-0.033	-1059.58	2147.16
		(0.010)	(0.079)	(0.034)	(0.017)	(4.860)	(0.009)	(0.013)	(0.096)	(0.024)	(0.001)	(0.001)	(0.004)	(0.001)	(0.040)		
	2	0.057	0.168	0.068	0.080	33.449	0.066	0.019	0.318	0.066		0.005	0.023		-0.033	-1059.58	2143.16
		(0.010)	(0.079)	(0.034)	(0.017)	(4.860)	(0.009)	(0.013)	(0.096)	(0.024)		(0.001)	(0.004)		(0.040)		
	3	0.013	0.712	0.135	0.021		0.068	0.000	0.322	0.082		0.003	0.022	0.030	0.113	-1076.99	2177.98
		(0.010)	(0.097)	(0.069)	(0.007)		(0.008)	(0.001)	(0.079)	(0.019)		(0.001)	(0.008)	(0.008)	(0.055)		

Table: Estimates of the bivariate Hawkes-POT models used for the analysis of the cluster behavior for extreme events of negative log-returns in stock markets and positive level-changes in IV indices, ending in December 31, 2012. Standard errors are in parenthesis.

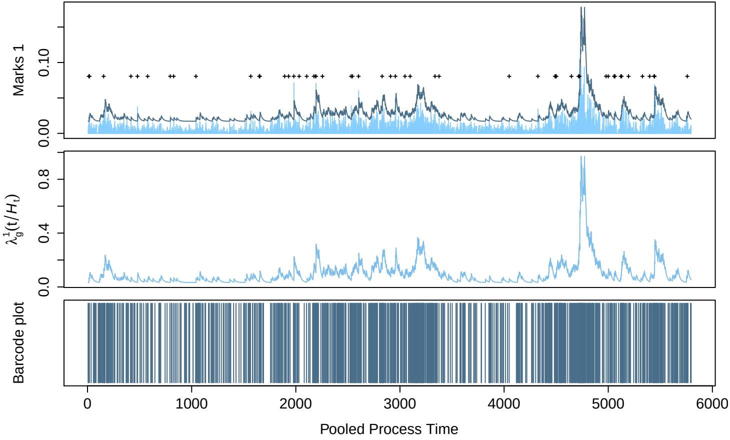
VaR in Sample

Univariate Hawkes Model I: $\lambda_g\left(t \mid \mathscr{H}_t\right) = \mu + \eta \sum_{i:t_i<t} \gamma e^{-\gamma(t-t_i)}$



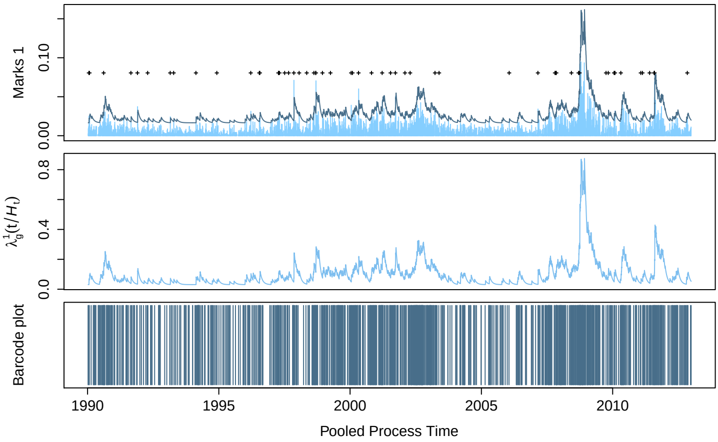
VaR in Sample

Univariate Hawkes Model II: $\lambda_g\left(t \mid \mathscr{H}_t\right) = \mu + \eta \sum_{i:t_i<t} e^{\delta w_i} \gamma e^{-\gamma(t-t_i)}$



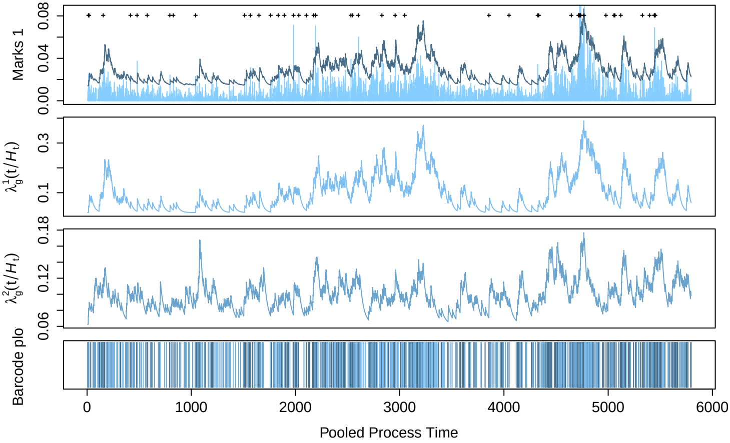
VaR in Sample

Univariate Hawkes Model III: $\lambda_g\left(t \mid \mathscr{H}_t\right) = \mu + \eta \sum_{i:t_i<t} e^{\delta w_i + \rho z_i} \gamma e^{-\gamma(t-t_i)}$



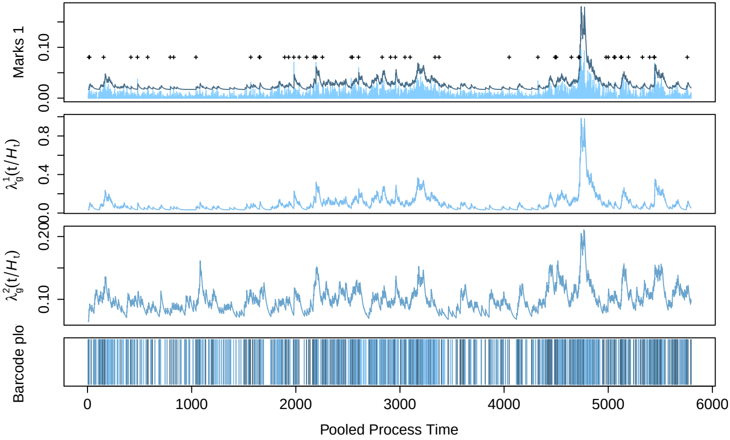
VaR in Sample

Bivariate Hawkes Model I:



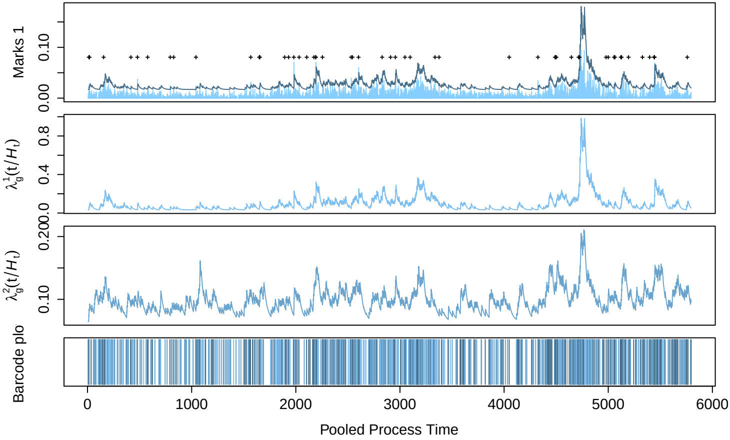
VaR in Sample

Bivariate Hawkes Model II:



VaR in Sample

Bivariate Hawkes Model III:



VaR Forecasting

		S&P500 -VIX					DAX - VDAX				
	α	Exc.	LRuc	LRind	LRcc	DQhit	Exc.	LRuc	LRind	LRcc	DQhit
EGARCH+IV	0.95	10	0.00	0.52	0.00	0.53	25	1.00	0.16	0.36	0.17
	0.99	0	0.00	1.00	0.01	1.00	5	1.00	0.75	0.95	0.75
	0.999	0	0.32	1.00	0.61	1.00	0	0.32	1.00	0.61	1.00
Hawkes-POT (M1)	0.95	15	0.03	0.34	0.05	0.35	16	0.05	0.30	0.08	0.32
	0.99	2	0.12	0.90	0.30	0.90	3	0.33	0.85	0.61	0.85
	0.999	0	0.32	1.00	0.61	1.00	0	0.32	1.00	0.61	1.00
Hawkes-POT (M2)	0.95	15	0.03	0.34	0.05	0.35	16	0.05	0.30	0.08	0.32
	0.99	2	0.12	0.90	0.30	0.90	3	0.33	0.85	0.61	0.85
	0.999	0	0.32	1.00	0.61	1.00	0	0.32	1.00	0.61	1.00
Hawkes-POT (M3)	0.95	15	0.03	0.34	0.05	0.35	13	0.01	0.40	0.02	0.42
	0.99	2	0.12	0.90	0.30	0.90	3	0.33	0.85	0.61	0.85
	0.999	0	0.32	1.00	0.61	1.00	0	0.32	1.00	0.61	1.00
Biv. Hawkes-POT (M1)	0.95	18	0.13	0.25	0.16	0.26	20	0.28	0.20	0.24	0.21
	0.99	1	0.03	0.95	0.09	0.95	3	0.33	0.85	0.61	0.85
	0.999	0	0.32	1.00	0.61	1.00	0	0.32	1.00	0.61	1.00
Biv. Hawkes-POT (M2)	0.95	19	0.20	0.22	0.20	0.23	20	0.28	0.20	0.24	0.21
	0.99	3	0.33	0.85	0.61	0.85	3	0.33	0.85	0.61	0.85
	0.999	0	0.32	1.00	0.61	1.00	0	0.32	1.00	0.61	1.00
Biv. Hawkes-POT (M3)	0.95	19	0.20	0.22	0.20	0.23	20	0.28	0.20	0.24	0.21
	0.99	3	0.33	0.85	0.61	0.85	4	0.64	0.80	0.87	0.80
	0.999	0	0.32	1.00	0.61	1.00	2	0.11	0.90	0.28	0.90

Table: Backtesting accuracy test results for the GARCH and Hawkes-POT models proposed, from January 2, 2012 to December 31, 2013.

Outline

Conclusions

- The role of implied volatility (IV) for forecasting the risk of extreme events in the form of VaR.
- This paper proposes a number of novel MPP models that include IV (univariate and bivariate)
- The empirical analysis: Major equity market indices and their associated IV indices.
- **In-sample results**: all of the models generate accurate VaR estimates that adequately pass a range of tests.

Conclusions

- **Forecasting**: to 1-day ahead prediction of VaR.
    - GARCH style models that include IV generate inaccurate forecasts of VaR and fail a number of tests relating to the rejection frequency of the VaR predictions.
    - Univariate MPP models provide more accurate forecasts with shortcomings at less extreme levels of significance.
    - The bivariate models that include the extreme IV events produce the most accurate forecasts of VaR across the full range of levels of significance.
- **The take-home message**: while IV is certainly of benefit for predicting extreme movements in equity returns, the framework within which it is used is important.
- The bivariate MPP model proposed here leads to superior forecasts of extreme risk in a VaR context.

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