

Extreme dependence with asymmetric thresholds: Evidence for the European Monetary Union

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Abstract

Existing papers on extreme dependence use symmetrical thresholds to define simultaneous stock market booms or crashes such as the joint occurrence of the upper or lower one percent return quantile in both stock markets. We show that the probability of the joint occurrence of extreme stock returns may be higher for asymmetric thresholds than for symmetric thresholds. We propose a non-parametric measure of extreme dependence which allows capturing extreme events for different thresholds and can be used to compute different types of extreme dependence. We find that extreme dependence among the stock markets of ten initial EMU member countries, the United Kingdom, and the United States is largely asymmetrical in the pre-EMU period (1989 to 1998) and largely symmetrical in the EMU period (1999 to 2010). Our findings suggest that ignoring the possibility of asymmetric extreme dependence may lead to an underestimation of the probability of co-booms and co-crashes.

JEL classification: C14; F30; G15

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1. Introduction

The increasing integration of financial markets worldwide suggests an increase in the comovement of stock returns since international capital flows have increased and lower transaction costs make cross-border arbitrage less costly. Since the degree of comovement between different stock markets has important implications for international diversification strategies and risk management, an interesting literature which has emerged deals with measuring comovement in stock returns.

This literature uses copulas to model comovement in stock returns. A copula is a function that models the connection between marginal distributions in order to restore the joint distribution. Compared to the correlation coefficient, copulas have the advantage of allowing us to model the dependence structure of different parts of the return distribution separately, particularly the tails of the return distribution. Moreover, an important drawback of using the correlation coefficient to measure comovement is that it is not informative about the degree of dependence in the tails of the underlying distribution and consequently underestimates the risk of extreme events such as joint stock market crashes (Embrechts et al., 2002; Embrechts, 2009). In order to avoid the methodological problems associated with the correlation coefficient, several interesting papers use extreme value theory to measure comovement in stock returns (see, for example, Longin and Solnik, 2001; Hartmann et al., 2004; Poon et al., 2004; McNeil and Frey, 2000; De Vries, 2005; Caillault and Guegan, 2005; Rodriguez, 2007; Straetmans et al., 2008; Hu, 2010; Beine et al., 2010; Aloui et al., 2011; Chollete et al., 2011; Bedendo et al., 2010; Hilal et al., in press (forthcoming) and Garcia and Tsafack, in press (forthcoming)).

The literature has so far focused on symmetrical thresholds to calculate measures of tail dependence, for example the one percent quantile of lowest/highest stock returns for both stock markets. This approach is intuitive in that simultaneous booms or crashes are defined by happening when two stock markets experience equal gains or losses. The most important drawback of using symmetric thresholds is, however, that (due to different risk-return characteristics, for example) simultaneous extreme stock market movements may be more likely when asymmetric thresholds are considered. For purposes of risk management, considering asymmetrical thresholds may therefore

be of particular interest in order to quantify the risk of simultaneous booms or crashes.

We contribute to the literature by studying asymmetric dependence structures in extreme stock returns. Contrary to existing papers that focus on the diagonal of the positive quadrant of a two dimensional space, i.e. on extremes around the angle $\pi/4$, we analyze the asymptotic extreme dependence also for asymmetric angles such as $\pi/8$ and $3\pi/8$ ¹. Using an extreme dependence measure with flexible thresholds, we show that extreme dependence in stock returns may be more likely for asymmetric than for symmetric thresholds. In order to estimate asymmetric extreme dependence, we use a non-parametric framework. More specifically, we use the so called canonical dependence function, introduced by Hsing et al. (2004) and embed it into a more general framework to calculate another measure of extreme dependence: the canonical conditional expectation measure of co-crashes or co-booms. The canonical conditional expectation measure is introduced by Huang (1992). Some interesting studies use this measure to quantify extreme dependence (for example, Straetmans, 2000; Straetmans et al., 2008; De Vries, 2005; Geluk et al., 2007). The estimated probabilities of these papers are based on the so-called stable tail dependence function introduced by Xin (1992). The main difference between this methodology and the approach introduced by Hsing et al. (2004) is that the canonical dependence function allows us to capture the complete extreme dependence in any direction and can easily be extended to higher dimensions. Moreover, it allows us to visualize the level of extreme tail dependence, a topic which has not yet received much attention.

We apply our framework to study possible asymmetries in extreme dependence among the stock markets of ten initial European Economic and Monetary Union (EMU) member countries, the United Kingdom, and the United States before (1989 to 1998) and after the introduction of the euro (1999 to 2010) using daily data. We find that simultaneous crashes and booms are largely asymmetrical in the pre-EMU period and largely symmetrical in the EMU period. Our finding suggests that a quantification of extreme stock market dependence, which relies solely on the special case of symmetrical thresholds, may underestimate the probability of extreme stock market

¹In the literature, the term asymmetrical tail dependence or extreme dependence is used to describe the result that upper tail dependence (joint stock market booms) may be less or more likely than lower tail dependence (joint stock market crashes). We use the term asymmetrical to describe the phenomenon that the degree of extreme dependence may be higher when different thresholds for two stock markets are used than when equal thresholds are employed.

events and therefore result in sub-optimal portfolios which do not achieve the maximum amount of diversification benefits. Moreover, we find that portfolios with larger values of extreme dependence experience larger stock returns, at least for the pre-EMU period. This finding is consistent with modern portfolio theory and suggests that higher returns compensate investors for the higher risk of extreme dependence of stock markets. Our portfolio analysis reveals that the level of asymmetries is lower for portfolios consisting of a large number of stock markets, which suggests that investors can diminish the risk associated with asymmetrical tail dependence by augmenting the portfolio size. Augmenting the portfolio size is however also associated with higher levels of stock market comovement which demonstrates the limits of portfolio diversification. Our results indicate that after the introduction of the euro co-booms and co-crashes have become much more symmetrical among EMU member countries. This result may be due to structural changes in the economic zone of the EMU member countries after the introduction of the euro such as the common monetary policy leading to symmetrical interest rate and exchange rate shocks in the EMU, more synchronized business cycles, and a higher integration of European stock markets. A possible extension to this study would be to analyze asymmetries in extreme dependence for other regions that did not experience such structural changes. Overall, we find that extreme dependence (symmetric and asymmetric) has increased among the stock markets of the EMU member countries following the formation of the EMU, which may be due to larger cross-border capital flows, more synchronized business cycles, a reduction of financial regulation, new electronic trading systems, and innovations in information technology. We do not find significant differences between upper and lower tail dependence which resembles the mixed findings on this issue in the literature.

The rest of the paper is organized as follows. Section 2 reviews the literature on extreme dependence. Section 3 describes the methodology. Section 4 presents the empirical application of our approach. Section 5 concludes.

2. Literature

Several interesting papers use extreme value theory in order to quantify the extreme dependence in stock markets. By avoiding the methodological drawbacks associated with the correlation coefficient, these studies reveal valuable insights into the nature of comovement of stock

markets for various countries and time periods. Longin and Solnik (2001) find that stock markets comove in bear markets but not in bull markets. Hartmann et al. (2004) introduce a measure for the probability of a joint crash in two stock markets. They find that joint crashes are significantly more likely than a multivariate normal distribution would suggest. What is more, they find that two stock markets co-crash in one out of five to six crashes. Poon et al. (2004) propose a novel tail dependence measure to study stock market comovement. They find that tail dependence for extremely low stock returns is more probable than for extremely high stock returns and that tail dependence increases in time, particularly for European countries. McNeil and Frey (2000) use a tail dependence measure to estimate value at risk models and show that models that rely on the normal distribution fail to estimate losses during financial crises. De Vries (2005) uses extreme value theory to study the risk of systemic bank breakdowns. He finds that, since the cash flow of banks depends on fat-tailed asset returns, systemic breakdowns are much more probable than the normal distribution suggests. Caillault and Guegan (2005) use Student and Archimedean copulas to analyze differences in upper and lower tail dependence among the Indonesian, Malaysian, and Thai stock markets in the period 1987 to 2002. For the Thailand-Indonesia and the Malaysia-Indonesia stock market pairs they find that lower tail dependence is more pronounced than upper tail dependence while no significant difference is found for the Thailand-Malaysia pair. Rodriguez (2007) uses a regime switching copula approach to study possible changes in tail dependence during the 1994/95 Mexican crisis and the 1997/98 Asian crisis. He finds evidence of significant increases of tail dependence by the Asian economies, while no change in tail dependence is found for the Latin American economies.

Straetmans et al. (2008) study the impact of the terrorist attacks of 9/11 on sectoral stock market comovement in the United States. They find that comovement of sectoral stock indices with respect to the NYSE Composite increased after 9/11, particularly within the sectors mostly affected by the terrorist attacks such as insurance, banking and finance, airline, and oil industries. Moreover, they find that comovement with respect to oil prices increased for all sectoral stock indices. Straetmans et al. (2008) argue that the higher degree of comovement may be due to a higher risk of terrorist attacks reflected in stock returns. Hu (2010) uses a copula approach to study the comovement between the Chinese and other major stock markets. Hu finds that the degree of

comovement of extremely high stock returns is much higher than comovement of extremely low stock returns. Beine et al. (2010) use a quantile regression approach to study the size and determinants of extreme dependence among the stock markets of 17 industrial economies for the period 1974 to 2006. They find an asymmetric effect of financial integration on tail dependence. While a higher degree of financial liberalization and lower exchange rate volatility increase comovement of extremely low stock returns, financial liberalization and exchange rate volatility have no impact on the comovement of extremely high stock returns. Bedendo et al. (2010) test how different choices for the dependence function can affect the prices of a set of multiasset equity options and the relevance of the dependence specification over both volatile and quiet market scenarios. They conduct the analysis for various 5-dimensional baskets of UK shares, and a wide range of payoffs for the multiasset options, consistent with the instruments traded on the market. They find that, in most circumstances, the choice of a dependence structure richer than the standard linear correlation does not seem to affect option prices substantially. However, the dependence function becomes more relevant in particularly volatile market condition. Aloui et al. (2011) use a multivariate copula approach (Gumbel, Galambos, and Husler-Reiss copulas) to study the extreme dependence between the stock markets of Brazil, China, India, Russia, and the United States for the period 2004 to 2009. They find that the degree of extreme comovement is larger in bullish than in bearish stock markets. Moreover, they find that extreme dependence with the United States stock market is higher for the commodity-dependent economies (Brazil and Russia) than for the finished products-exporting economies (China and India), which suggests that an economy's economic structure impacts extreme dependence. Chollete et al. (2011) use normal, Gumbel, and Student-t copulas to investigate extreme dependence among the stock markets of East Asia, Latin America, and the G5 countries in the period 1990 to 2006. They find that extreme dependence has generally increased over time and that the Latin American stock markets feature a greater downside risk than the G5 and East Asian stock markets. Moreover, Chollete et al. (2011) find that stock markets with the least diversification benefits, i.e. with the greatest degree of extreme dependence, do not generally compensate investors with large returns. Garcia and Tsafack (in press) (forthcoming) use a regime switching copula model to study extreme dependence in international stock and bond markets. Their model allows for regime switches from a normal regime

with linearly correlated returns to a regime where markets commove only at extreme negative returns. Using data for the United States-Canada and the Germany-France market pairs in the period 1985-2004 they find that dependence between two equity markets or two bond markets is strong in both regimes whereas the dependence for stock-bond markets pairs is weak. Hilal et al. (in press) (forthcoming) use an extreme dependence framework to test the VIX futures' ability of hedging instruments for the S&P 500 lower tail risk. Using daily data for the period 2004-2008 they show that their simulated hedging portfolio outperforms five alternative hedging measures.

3. Methodology

The multivariate extreme value theory and its application to financial problems is a relatively new but rapidly growing field.² In order to make some inference about the tail of a distribution, a fair methodology to proceed with is to approximate the tail asymptotically as an extreme value distribution. However, there is no finite dimensional parametrization for all classes of dependence structure in a multivariate problem. On the other hand, it is well known that the dependence of a multivariate distribution, in this case a multivariate extreme distribution, can be characterized by means of a copula function.

In this section we introduce a nonparametric approximation to the extreme copula function, which will provide us with an effective method to estimate and visualize measures of extreme dependence. For ease of the exposition we consider the bivariate case.

Let $\mathbf{X} = \{X_{n1}, X_{n2}\}$ be an iid bivariate random vector with common distribution function F with continuous marginals. Suppose that there exist norming constants a_1, a_2 and $b_1, b_2 \in \mathbb{R}^+$ such that F converges to a limit distribution with non-degenerate marginals

$$G(x_1, x_2) = \lim_{n \rightarrow \infty} F^n(M_{n1} \leq x_1 b_1 + a_1, M_{n2} \leq x_2 b_2 + a_2), \quad (1)$$

where $M_{n1} = \bigvee_{i=1}^n X_{i1}$ and $M_{n2} = \bigvee_{i=1}^n X_{i2}$ describe the vector of componentwise maxima. In addition, the univariate marginal distribution functions of each marginal $G_1(x_1)$ and $G_2(x_2)$ are also extreme value distributions.

²See McNeil et al. (2005) for an interesting introduction to this subject.

Note, that the limit distribution (1) can be written as

$$\lim_{n \rightarrow \infty} (1 - F(x_1 b_1 + a_1, x_2 b_2 + a_2)) = -\ln G(x_1, x_2). \quad (2)$$

Since these are monotone functions, a continuous version of the limit remains the same, replacing $n \in \mathbb{N}$ by $t \in \mathbb{R}_+$. Based on this argument Einmahl et al. (2001) propose an alternative representation of (2) in terms of polar coordinate such as follows

$$\begin{aligned} G(x_1, x_2) &= \exp\{-\Lambda(x_1, x_2)\} \\ &= \exp\left\{-\int_0^{\pi/2} \left(\frac{1 \wedge \tan \varphi}{x_1} \vee \frac{1 \wedge \cot \varphi}{x_2}\right) \Phi(d\varphi)\right\}, \end{aligned} \quad (3)$$

where $x \wedge y$, $x \vee y$ indicate the minimum and the maximum between x and y , respectively. Further, Φ is a finite measure on $[0, \pi/2]$, with the condition that

$$\int_0^{\pi/2} (1 \wedge \tan \varphi) \Phi(d\varphi) = \int_0^{\pi/2} (1 \wedge \cot \varphi) \Phi(d\varphi) = 1.$$

The intensity measure $\Lambda(x_1, x_2)$ defines the dependence among the marginals and therefore the success probability of extreme events. Indeed, $\Lambda(x_1, x_2)$ is a version of the classical limit theorem of Poisson. Observe that $\Lambda(x_1, x_2)$ could be derived directly from (1) due to the fact that this measure is equal to the right-hand side of this relation:

$$\begin{aligned} \Lambda(x_1, x_2) &= \lim_{t \rightarrow \infty} tP\left(G_1(X_1) > 1 - \frac{x_1}{t} \text{ or } G_2(X_2) > 1 - \frac{x_2}{t}\right) \\ &= \lim_{t \rightarrow \infty} tP\left(t(\bar{G}_1(X_1), \bar{G}_2(X_2)) \in \mathbf{A}\right), \end{aligned} \quad (4)$$

where $\bar{G}_j = 1 - G_j$ and $\mathbf{A} = ([x_1, \infty] \times [x_2, \infty])^c$.

An intuitive estimator of this measure is its empirical counterpart. Remember that the empirical distribution given by $\hat{G}_j(X_{ij}) = \frac{1}{n} R_i^{X_j} = \frac{1}{n} \text{rank}(-X_{i1})$ is the standard estimator for the distribution function G_j for iid observations.

An estimator for $\Lambda(x_1, x_2)$ is then given by

$$\begin{aligned}\tilde{\Lambda}_n(x_1, x_2) &= \lim_{t \rightarrow \infty} t \hat{P}_n \left(t \left(\bar{G}_1(X_1), \bar{G}_2(X_2) \right) \in \mathbf{A} \right) \\ &= \lim_{t \rightarrow \infty} \frac{t}{n} \sum_{i=1}^n \mathbf{1}_{\mathbf{A}} \left(\bar{G}_1(X_1), \bar{G}_2(X_2) \right),\end{aligned}$$

where $\mathbf{1}_{\mathbf{A}}$ is an indicator function for the events that belong to \mathbf{A} . Writing $\tau = t/n$ we obtain:

$$\tilde{\Lambda}_\tau(x_1, x_2) = \tau \sum_{i=1}^n \mathbf{1}_{\mathbf{A}} \left(\tau \left(R_i^{X_1}, R_i^{X_2} \right) \right).$$

The extension of this measure to higher dimensions is straightforward. This estimator was introduced by Hsing et al. (2004) in order to estimate other intuitive measures of dependence in higher dimensions.

3.1. The canonical conditional expectation measure of extreme dependence

An alternative characterization of the extreme dependence in a multivariate framework can be presented using the canonical tail function proposed by Hsing et al. (2004), which is motivated by the representation of Einmahl et al. (2001) in (3) for the bivariate case. A feature of $\Lambda(x_1, x_2)$ is its homogeneity of degree one. The fact that for instance $\Lambda(x_1, x_2) = \Lambda(1, x_2/x_1)$ motivates the following definition.

Definition 1. Let $\mathbf{X} = \{X_1, X_2\}$ be an iid bivariate random vector such that (1) holds.

Then, provided that the limits exist, the canonical tail function is given by

$$\begin{aligned}\psi(\varphi) &= \lim_{\substack{x_1, x_2 \rightarrow \infty \\ \bar{G}_2(x_2)/\bar{G}_1(x_1) \rightarrow \cot \varphi}} \frac{1 - P(X_1 \leq x_1, X_2 \leq x_2)}{P(X_1 > x_1)}, \quad 0 < \varphi < \pi/2 \\ &= \Lambda(1, \cot \varphi)\end{aligned}$$

or alternatively in terms of copulas as

$$\psi(\varphi) = \lim_{\substack{u_1, u_2 \rightarrow 1 \\ (1-u_2)/(1-u_1) \rightarrow \cot \varphi}} \frac{1 - C(u_1, u_2)}{1 - u_1}.$$

This function only depends on the angle φ and measures the dependence in any direction of the positive quadrant of a bivariate distribution, including the main diagonal $\pi/4$.

Its empirical counterpart is given by

$$\begin{aligned} \tilde{\psi}_\tau(\varphi) &= \tilde{\Lambda}_\tau(1, \cot \varphi) \\ &= \tau \sum_{i=1}^n \mathbf{1}_{\mathbf{A}}(\tau(R_i^{X_1}, R_i^{X_2}) \in \mathbf{A}) \\ &= \tau \sum_{i=1}^n \mathbf{1}_{\mathbf{A}}(R_i^{X_1} \leq \tau^{-1} \text{ or } R_i^{X_2} \leq \tau^{-1} \cot \varphi), \quad 0 < \varphi < \pi/2. \end{aligned} \tag{5}$$

The multivariate version of this measure is straightforward. Hsing et al. (2004) define the set \mathbf{A} to measure tail dependence in a multivariate frameworks as follows

$$\mathbf{A}_{\varphi_1, \dots, \varphi_{d-1}} := \{(x_1, \dots, x_d) \in [0, \infty]^d : x_1 \wedge x_2 \tan \varphi_1 \wedge \dots \wedge x_d \tan \varphi_{d-1} \leq 1\},$$

which has a clear geometric interpretation. This yields in combination with the definition of the measure Λ to

$$\psi(\varphi_1, \dots, \varphi_{d-1}) := \Lambda(\mathbf{A}_{\varphi_1, \dots, \varphi_{d-1}}) = \Lambda(1, \cot \varphi_1, \dots, \cot \varphi_{d-1}), \tag{6}$$

with the empirical counterpart

$$\tilde{\psi}_\tau(\varphi_1, \dots, \varphi_{d-1}) = \tau \sum_{i=1}^n \mathbf{1}_{\mathbf{A}} \left(R_i^{X_1} \leq \tau^{-1} \text{ or } \bigcup_{j=2}^d R_i^{X_j} \leq \tau^{-1} \cot \varphi_{j-1} \right). \tag{7}$$

The estimation of ψ requires choosing an appropriate parameter τ . The selection of this parameter is directly related to the definition of an extreme observation, i.e., one has to decide which events of the complete data set belong to the subset relevant for the estimation of ψ . The quality of

the estimation results for ψ from (7), the convergence properties of (5) and its multivariate extension depend on the choice of τ . At the same time, the selection of τ is equivalent to the estimation of a probability based on the $1/\tau$ largest values of a bivariate vector of observations. This problem is called the threshold selection problem in extreme value theory (see, for example, Embrechts et al., 1997 for more reference).

In practice, the basic idea is to select a value for $1/\tau$ that is a good compromise between a potential bias (due to the only approximate validity of the extreme value theory background), and the variance of the estimate which strongly depends on the size of the selected subset of extreme events. In order to address this trade-off associated with the selection of τ , we take advantage of the homogeneity property of ψ as explained in the following.

Proposition 2. *For all angles $\varphi_j \in [0, \pi/2]$ we have*

1. The canonical tail function $\psi(\varphi_1, \dots, \varphi_{d-1})$ is a monotone decreasing function bounded by $\psi_{dep}(\varphi_1, \dots, \varphi_{d-1}) \leq \psi(\varphi_1, \dots, \varphi_{d-1}) \leq \psi_{ind}(\varphi_1, \dots, \varphi_{d-1})$, where

$$\psi_{dep}(\varphi_1, \dots, \varphi_{d-1}) = 1 \vee \cot \varphi_1 \vee \dots \vee \cot \varphi_{d-1}$$

and

$$\psi_{ind}(\varphi_1, \dots, \varphi_{d-1}) = 1 + \cot \varphi_1 + \dots + \cot \varphi_{d-1}.$$

2. The tail function is homogeneous $\alpha\psi(\varphi_1, \dots, \varphi_{d-1}) = \psi(\alpha\varphi_1, \dots, \alpha\varphi_{d-1})$ for all $\alpha > 0$.
3. For all $a_j > 0$ with $j = 1, \dots, d-1$ we have

$$\left(1 \wedge \bigwedge_{j=1}^{d-1} a_j\right) \psi(\varphi_1, \dots, \varphi_{d-1}) \leq \psi(a_1\varphi_1, \dots, a_{d-1}\varphi_{d-1}) \leq \left(1 \vee \bigvee_{j=1}^{d-1} a_j\right) \psi(\varphi_1, \dots, \varphi_{d-1}).$$

4. For $d = 2$

$$\psi_{X_1X_2}(\varphi) = \cot \varphi \psi_{X_1X_2}(\pi/2 - \varphi).$$

Given the homogeneity property 2 in Proposition 2, we choose τ in the empirical application so

that ψ_τ mimics this property for any parameter $u > 0$. Thus, for a fixed τ , we graph

$$\left\{ \frac{\tilde{\psi}_\tau(u\varphi_1, \dots, u\varphi_{d-1})}{u\tilde{\psi}_\tau(\varphi_1, \dots, \varphi_{d-1})}, u > 0 \right\}. \quad (8)$$

The idea is that the ratio should be roughly constant and close to 1 for any $u > 0$, when τ is chosen adequately. The plots will look differently for different values of τ . Therefore we will choose the τ for which the homogeneity property is more evident. In the practical application we replace $\psi(\varphi_1, \dots, \varphi_{d-1})$ by its empirical counterpart $\tilde{\psi}_\tau(\varphi_1, \dots, \varphi_{d-1})$.

In order to demonstrate the choice of τ empirically we use the following example of the stock market pair Germany and France for the pre-EMU period and examine the lower tail dependence. We pick a value of τ so that ψ mimics the homogeneity property. In Figure 1 we illustrate the ratio of Eq. (8) on the y-axis and the range u on the x-axis from 0.1 to 5.0, the interval where we hope that the homogeneity property holds³.

We simulate this ratio for four values of τ (0.01, 0.015, 0.02, and 0.025)⁴. The results indicate that values of τ between 0.01 and 0.015 seem to be a good choice because the values for the ratio of Eq. (8) fluctuate around the horizontal line at the value 1, which means that $\tilde{\psi}_\tau(u\varphi) = u\tilde{\psi}_\tau(\varphi)$ holds. Thus, we use a value of $\tau = 0.01$ to estimate our CCEM measure since the homogeneity property holds.

However, the canonical tail function has the drawback that it is not a global measure of extreme dependence.

Huang (1992) proposed a more general measure of tail dependence, *the conditional expectation measure*. It is defined as the conditional expected value of the probability of that k extreme events occur in d -marginals. For instance, let k be the number of extreme events that simultaneously occur in 2-dimensions. In a finite sample framework, we have the following expression for the conditional expected value of the probability that $k \leq 2$ extreme events occur, given that at least one $k \geq 1$ extreme event happened. From elementary probability theory we have that

³From a theoretical point of view the homogeneity property should hold for each value of u in the interval $(0, \infty]$. However, because the results are obtained asymptotically in a finite sample, we defined arbitrary a small interval for the range u where the homogeneity property holds

⁴In our empirical applications we examine much more than four possible values of τ than done in the example.

$$\begin{aligned}
E\{k \mid k \geq 1\} &= \lim_{x_1, x_2 \rightarrow \infty} \frac{P(X_1 \leq x_1, X_2 > x_2) + P(X_1 > x_1, X_2 \leq x_2)}{1 - P(X_1 \leq x_1, X_2 \leq x_2)} \\
&\quad + \frac{2P(X_1 > x_1, X_2 > x_2)}{1 - P(X_1 \leq x_1, X_2 \leq x_2)} \\
&= \frac{P(X_1 > x_1) + P(X_2 > x_2)}{1 - P(X_1 \leq x_1, X_2 \leq x_2)} \\
&= \frac{E\{k\}}{P\{k \geq 1\}}.
\end{aligned}$$

This linkage measure has the advantages that it can be easily extended to more dimensions. Moreover, one does not need to condition on a specific marginal, whereby one would look only into one direction in the plane of extreme events. For this reason, different authors have used this measure of extreme dependence. For instance, Straetmans (2000) uses this measure to assess extremal spillovers in equity markets. Hartmann et al. (2004) use it to investigate asset market linkages in crisis periods. De Vries (2005) uses it to calculate the potential for systemic breakdowns in banks and finally, Geluk et al. (2007) propose this measure as an index of financial fragility.

In order to study possible asymmetries in extreme dependence, we extend this measure to the framework of this paper. That is, we do not restrict our analysis to symmetric thresholds but allow asymmetrical thresholds, i.e., different directions in terms of $\psi(\varphi_1, \dots, \varphi_{d-1})$ as explained in the following.

Definition 3. Let $\mathbf{X} := \{X_1, \dots, X_d\}$ be a d -variate vector of iid non-negative random variables with distribution $G \in \mathbb{R}^d$, and thresholds $x_1, \dots, x_d \rightarrow \infty$ such that $\frac{\bar{G}(x_j)}{\bar{G}(x_1)} \rightarrow \cot \varphi_{j-1}$ for all $j = 2, \dots, d$. The d -variate *canonical conditional expectation measure (CCEM) of extreme dependence* is given by

$$E\{k \mid \varphi_1, \dots, \varphi_{d-1}, k \geq 1\} = \frac{1 + \sum_{j=1}^{d-1} \cot \varphi_j}{\psi(\varphi_1, \dots, \varphi_{d-1})}. \quad (9)$$

In the case that the random vector pair is completely independent, $E\{k \mid \varphi_1, \dots, \varphi_{d-1}, k \geq 1\}$ reaches its lower bound equal to 1, which implies that the data are also asymptotically independent.

On the other hand, if the data are completely dependent, then $E\{k \mid \varphi_1, \dots, \varphi_{d-1}, k \geq 1\}$ will equal d , i.e., 2 in the bivariate case. Notice that $E\{k \mid \varphi_1, \dots, \varphi_{d-1}, k \geq 1\} = E\{k \mid k \geq 1\}$ for $\varphi_1 = \dots = \varphi_{d-1} = \pi/4$. Observe that $E\{k \mid \varphi_1, \dots, \varphi_{d-1}, k \geq 1\}$ is the upper bound of the canonical tail function, the independent case, divided by the observed canonical tail function.

In the following, we will use an example to illustrate that the maximum of this measure does not necessarily occur for a symmetric threshold when $x_1 = \dots = x_d \rightarrow \infty$, or equivalently when $\varphi_1 = \dots = \varphi_{d-1} = \pi/4$.

Example. Define the following three processes $X_1 = \xi_i$, $X_2 = \alpha\xi_i \vee (1 - \alpha)\xi_{i+1}$, and $X_3 = \beta\xi_i \vee (1 - \beta)\xi_{i+2}$, where ξ is iid random vector with Frechet distribution, i.e, $P(\xi < x) = \exp(-\frac{1}{x})$ and $0 < \alpha, \beta < 1$. Clearly the three processes are in some degree extreme dependent, which is due to the factors α and β . Moreover, note that X_1, X_2 and X_3 have Frechet marginals.

We will estimate the the canonical tail functions $\psi_{X_1X_2}$ and $\psi_{X_2X_3}$. To this end, we choose thresholds x_2, x_3 which satisfy the Eq.(6). This can be done by replacing $x_2 = x_1 \tan \varphi$, $x_3 = x_1 \tan \varphi$ and evaluating in the limit when $x_1 \rightarrow \infty$. Then, for $\psi_{X_1X_2}$ we have:

$$\begin{aligned} \psi_{X_1X_2}(\varphi) &= \frac{1 - P(X_1 \leq x_1, X_2 \leq x_1 \tan \varphi)}{P(X_1 > x_1)} \\ &= \frac{1 - P(\xi_i \leq x_1 \wedge x_1 \alpha^{-1} \tan \varphi, \xi_{i+1} \leq x_1 (1 - \alpha)^{-1} \tan \varphi)}{P(X_1 > x_1)}, \end{aligned}$$

and given that ξ is an iid random vector with Frechet distribution we can derive:

$$\psi_{X_1X_2}(\varphi) = \frac{1 - \exp\left\{-\frac{1}{x_1} (1 \vee \alpha \cot \varphi + (1 - \alpha) \cot \varphi)\right\}}{1 - \exp\left(-\frac{1}{x_1}\right)}$$

By L'Hospital when $x_1 \rightarrow \infty$ we can derive:

$$\psi_{X_1X_2}(\varphi) = 1 \vee \alpha \cot \varphi + (1 - \alpha) \cot \varphi.$$

Following the same procedure for the pair (X_2, X_3) we obtain:

$$\psi_{X_2 X_3}(\varphi) = \alpha \vee \beta \cot \varphi + (1 - \alpha) + (1 - \beta) \cot \varphi.$$

Replacing these canonical tail functions in (9) we can estimate the CCEM of extreme dependence $E\{k \mid \varphi, k \geq 1\}$. For instance, we are interested in these measures for the values $\alpha = \beta = 1/2$. Applying (9) we obtain that

$$E\{k \mid \varphi, k \geq 1\}_{X_1, X_2} = (1 + \tan \varphi) \wedge \left(1 + \frac{1}{2 \tan \varphi + 1}\right),$$

and

$$E\{k \mid \varphi, k \geq 1\}_{X_2, X_3} = \frac{2(\cot \varphi + 1)}{1 \vee \cot \varphi + 1 + \cot \varphi},$$

for $\varphi \in [0, \pi/2]$.

We display both measures, $E\{k \mid \varphi, k \geq 1\}_{X_1, X_2}$ and $E\{k \mid \varphi, k \geq 1\}_{X_2, X_3}$, in Figure 2, where the solid line shows the theoretically derived values and the dashed line shows the estimation due to simulation. From Figure 2 it is clear that both variable pairs differ with respect to the symmetry of extreme dependence. For the first variable pair, X_1, X_2 , (see the left panel of Figure 2) the extreme dependence measure, $E\{k \mid \varphi, k \geq 1\}_{X_1, X_2}$, reaches its maximum at $\varphi^* = \arctan(0.5)$, i.e., it has an asymmetric extreme dependence structure, where joint crashes or co-booms are most probable for the asymmetric threshold $\varphi^* = \arctan(0.5)$, i.e., $E\{k \mid \varphi^*, k \geq 1\}_{X_1, X_2} = 1.5$. For the second variable pair, X_2, X_3 , (see the right panel of Figure 2) the extreme dependence measure is highest at the symmetric threshold $\varphi = \pi/4$. This simple example illustrates that pairs of variables, which follow different stochastic processes, can differ fundamentally with respect to the degree of symmetry of extreme dependence. Empirically, stock returns of different stock markets may also be described by different stochastic processes. We would thus also expect that asymmetric extreme dependence structures may play an important role in empirical applications as well.

Investors should take this type of asymmetries into account since they may affect diversification benefits of portfolios. Assume that an investor compares two portfolios, one with X_2, X_3 and other with X_1, X_2 . Figure 2 suggests that the risk of potential portfolio losses of portfolio X_1, X_2

triggered by joint crashes is the highest for an asymmetric threshold, whereas losses of portfolio X_2, X_3 , triggered by co-crashes, are most probable for the conventional symmetric threshold. Ignoring the possibility of co-crashes and co-booms at asymmetric thresholds may therefore lead to a significant underestimation of extreme dependence of portfolios, which may lead to the composition of inefficient portfolios and sub-optimal diversification benefits.

The numerical example shows that asymmetries can be present when estimating extreme dependence functions, i.e., that the CCEM measure of extreme dependence does not necessarily take its maximum value for the symmetrical boom/crash threshold (the angle $\varphi = \pi/4$). Several approaches may be used to detect possible asymmetries in extreme dependence empirically. One possible option would be to estimate the CCEM measure of extreme dependence for all possible values of the threshold, i.e., for all values of φ in the range $[0, \pi/2]$. In order to demonstrate this approach we calculate the CCEM measure of lower tail dependence for the stock market pair Germany and France for the pre-EMU period for this continuous range of $[0, \pi/2]$. Figure 3 illustrates the results⁵.

The results suggest that the maximum value of the CCEM measure of lower tail dependence is around $\varphi = 0.7$, i.e., at a lower level than the symmetrical value $\pi/4$. That is, the probability of simultaneous crashes of the German and French stock markets is highest for an asymmetrical crash threshold. Using the symmetrical crash threshold of $\varphi = \pi/4$ would thus underestimate the risk of simultaneous crashes.

In this paper, we study possible asymmetries in extreme dependence for 12 stock markets for lower and upper tail dependence and for the pre-EMU and the EMU period separately. Analyzing the full range of possible values for the thresholds φ would therefore not be feasible since the results (as done here for the Germany-France example) cannot be illustrated for all country pairs studied in the paper.

In this paper we use two approaches to study asymmetries in extreme dependence. In Section 4.2 we study extreme dependence for bilateral stock market combinations. For each stock market pair we estimate the CCEM measure of extreme dependence for the symmetrical threshold $\pi/4$ and

⁵In the estimation we use the value 0.01 for the parameter τ as outlined on p. 12.

two asymmetrical thresholds, $\pi/8$ and $3\pi/8$, and study whether the CCEM measure takes its largest value at the symmetrical threshold or an asymmetrical threshold. Although this simple approach does not allow us to determine for which exact threshold the extreme dependence measure takes its maximum value, it allows us to analyze in a comparable way whether or not there are asymmetries in extreme dependence.

In Section 4.3 we analyze possible asymmetries in extreme dependence using portfolios of stock markets. In this multivariate case the estimation of the CCEM measure of extreme dependence becomes intractable for all values of φ in the range $[0, \pi/2]$. Therefore we use a simulation where the asymmetrical values of φ are drawn randomly and the results of the CCEM measure for the asymmetrical thresholds are compared with the symmetrical case. Similar to the bivariate case, this application allows us to study whether or not there are asymmetries in extreme dependence in a portfolio setting.

3.2. Hypothesis testing

In the empirical application in Section 4, we test whether extreme dependence changes over time and whether there are differences between upper and lower tail dependence. In this section we introduce a test that can be used to assess whether the difference between two measures of extreme dependence are statistically significant. To test for significant differences between two measures of extreme dependence (CCEM), we use the following t -statistic for a given angle $\varphi \in (0, \pi/2)$:⁶ Let us denote $E_1 = E_1 \{k \mid \varphi, k \geq 1\}$ and $E_2 = E_2 \{k \mid \varphi, k \geq 1\}$, then the statistic

$$t = \frac{E_1 - E_2}{\sqrt{\hat{\sigma}^2(E_1) + \hat{\sigma}^2(E_2) - 2\widehat{cov}(E_1, E_2)}}, \quad (10)$$

converges to a standard normal distribution in large samples. The null hypothesis is equal CCEM, in this case $H_0 : E_1 \{k \mid \varphi, k \geq 1\} = E_2 \{k \mid \varphi, k \geq 1\}$. The asymptotic standard errors and covariance are obtained via a block bootstrap procedure. We opt for bootstrapping in blocks due to the fact that nonlinear dependencies might be present in the stock market returns. In order to obtain an

⁶The asymptotic normality of the canonical tail function can be derived by means of second order conditions. See De Haan and Ferreira (2006) and Schmidt and Stadtmueller (2006) for more reference.

optimal size for the block length, we follow the procedure proposed in Politis and White, 2004. We use a block size of 550.⁷

We further distinguish between two kinds of tests. The structural change test assesses the statistical significance in the change of a country pair's CCEM between two periods for the negative and positive returns separately. Increases in lower and upper extreme dependence over time are indicated by significantly negative t-values. We also test for significant differences between upper and lower extreme dependence for a country pair in a given period, where significantly positive (negative) values indicate that lower extreme dependence is significantly more pronounced (less pronounced) than upper extreme dependence.

4. Empirical Application

4.1. Data description and their statistics

We use data on daily closing stock indices from ten EMU member countries: Austria (ATX) [AT], Belgium (BRUSSELS) [BE], Finland (OMX HELSINKI) [FI], France (CAC 40) [FR], Germany (DAX) [DE], Ireland (ISEQ) [IE], Italy (Milan SE) [IT], the Netherlands (AEX) [NL], Portugal (PSI General) [PT], and Spain (Madrid SE) [ES]. Moreover, we include data on the United Kingdom (FTSE100) [UK] and the United States (S&P 500) [US] in order to investigate comovement with the stock markets of these important international financial centers. Data is taken from Datastream. Stock returns are computed using the daily log-difference in the U.S dollar-denominated stock index multiplied by 100. Data is sampled daily and covers the period January 2, 1989 to June 30, 2010. Non trading days are excluded from the sample. The time sample is split into two sub-periods: the period before the introduction of the euro (January 2, 1989 to December 31, 1998) and the period after the euro introduction (January 2, 1999 to June 30, 2010).

Table 1 presents some descriptive statistics for the stock index returns. The first row indicates the country abbreviation (listed above). All stock market indices have a positive mean in the first period, while in the second period the results are mixed. Most stock markets show a negative skewness in both periods, which indicates that extreme negative events are a common feature for

⁷More details on the block bootstrap procedure are available upon request.

all indices. The kurtosis is also significantly greater than for the normal distribution for all series in both periods. The results of the Ljung-Box test statistic indicate the presence of serial dependence (or autocorrelation) in all returns. Furthermore, the Engle (10) test statistic clearly indicates that ARCH effects are likely to be found in all stock market returns in both periods suggesting that the stock market returns exhibit conditional heteroskedasticity. Consequently, we fit an autoregressive process to all stock returns plus a general GARCH process to take account for the presence of serial dependence and heteroskedasticity.

The models employed for the marginal distributions of the stock markets are assumed to be completely characterized by an AR(1), Skew-t GARCH(1,1) specification for the two sample periods.⁸ For the estimation of the different measures of extreme dependence we use the standardized residuals of the GARCH models.

Before we employ the estimator (7) of ψ to calculate the different measure of extreme dependence, we select the optimal parameter τ . We solve this problem using the homogeneous property (8). Different values were tested for all stock return pairs under study. According to our results, $\tau = 0.01$ is found to be the optimal value.

4.2. Results for bilateral extreme dependence

In this section we present the results of our extreme dependence measure for the ten initial EMU member countries, the United Kingdom, and the United States before and after the introduction of the euro. Our canonical conditional expectation measure (CCEM) of extreme dependence is calculated according to Eq.(9) and measures (one plus) the probability that two stock markets co-crash (lower tail dependence) or co-boom (upper tail dependence) for a given crash or boom threshold of stock returns. Our extreme dependence measure is bounded between 1 (complete independence) and 2 (complete dependence). We showed in our simple example (see Example on pp. 14 to 15) that, depending on the assumptions of the stochastic process, the probability that two stock markets co-crash or co-boom can be higher for asymmetric thresholds than for symmetric thresholds. Therefore, taking asymmetric thresholds into consideration may also be important

⁸Detailed results of the parameter estimates and standard errors for the marginal distribution models for all stock index returns are available from the authors upon request.

when quantifying extreme dependence in order to estimate the diversification benefits of portfolios using real world financial data. An advantage of our extreme dependence approach is that although it nests symmetric thresholds as a special case, we also consider asymmetric thresholds. We estimate extreme dependence in three different directions: $\pi/8$, $\pi/4$ (the symmetrical case), and $3\pi/8$.

Table 2 reports the estimation results for the canonical conditional expectation measure (CCEM) of extreme dependence for all considered country pairs. The left column reports the results for lower tail dependence, i.e., the expected number of co-crashes. The right column shows the results for upper tail dependence, i.e., the expected number of co-booms. The extreme dependence measure is estimated for each threshold and for the pre-EMU period (1989 to 1998) and the EMU period (1999 to 2010) separately. We find that extreme dependence is largely asymmetrical in the pre-EMU period (1989 to 1998) but has become much more symmetrical in the EMU period (1999 to 2010). For the pre-EMU period, only 42% of the lower tail dependence country pairs and 3% of the upper tail dependence country pairs take the maximum value at the symmetrical threshold ($\pi/4$), while for the EMU period, 91% of the lower tail dependence country pairs and 52% of the upper tail dependence country pairs have their largest value at the symmetrical threshold. This result suggests that in the pre-EMU period co-booms and co-crashes were most probable for different/asymmetrical thresholds of the two countries' stock markets while in the EMU period co-booms and co-crashes occurred most probably at equal/symmetrical thresholds. Ignoring this finding that co-booms and co-crashes may most probably occur at asymmetrical thresholds may therefore lead to an underestimation of extreme stock market events and consequently a sub-optimal portfolio composition and a reduction in diversification benefits. The increase in the symmetry of extreme dependence among the EMU member countries after the introduction of the euro may be explained by several factors. First, since the introduction of the euro the ECB conducts a common monetary policy for all EMU member countries and thus interest rate shocks affect the required rate of return of stock market investors of all member countries in the same way, while in the pre-EMU period different national monetary policies and interest rate changes may have resulted in asymmetrical stock market comovement. In a similar vein, exchange rate shocks were more asymmetrical in the pre-EMU period whereas the introduction of the euro has resulted in

symmetrical exchange rate shocks for all EMU member countries which may contribute to the more symmetrical extreme dependence in the EMU period. Moreover, triggered by the common monetary policy and increasingly coordinated fiscal policies (due to the compliance with the Maastricht criteria), business cycles have become more synchronized after the introduction of the euro. Since shocks to economic fundamentals may induce extreme stock market movements, more symmetrical economic shocks in the EMU member countries may partly explain our finding that stock market comovement has become more symmetrical in the EMU period. The mergers of European stock exchanges after the introduction of the euro (for example, the merger of the Amsterdam, the Brussels, and the Paris stock exchange to form Euronext) and the establishment of transnational stock market trading platforms have increased the speed of cross-border equity flows which may also help to explain our finding that co-crashes and co-booms have become more symmetrical during the EMU period. For the UK and US country pairs we find that extreme dependence of the EMU member countries has become much more symmetrical with respect to these international financial centers after the introduction of the euro, whereby this effect is much more pronounced for the United Kingdom than for the United States. This finding suggests that the UK stock market has become much more integrated into the other European stock markets than the US stock market.

Table 3 reports the estimation results for the t-tests (see Eq.(10)) testing the null of no difference between two measures of extreme dependence. The first and second columns report the results of the t-test testing whether lower and upper tail dependence change significantly after the introduction of the euro. Values below -1.96 (above 1.96) indicate that extreme dependence becomes significantly larger (smaller) after the formation of the EMU at the 5% level of significance. The third and fourth columns report the t-values testing the null that upper and lower tail dependence are equal in the pre-EMU or the EMU period. Values below -1.96 indicate that upper tail dependence is significantly more pronounced than lower tail dependence and values above 1.96 indicate that lower tail dependence is significantly more pronounced than upper tail dependence in the respective period. The results in the first and second columns indicate that lower and upper tail dependence has significantly increased after the introduction of the euro, which resembles the frequent finding in the literature that stock market comovement has increased in the last decades.

This finding suggests that the stock markets of Europe and the main financial centers have become increasingly integrated, which may be due to larger cross-border capital flows, a reduction in regulation, new electronic trading systems, and innovations in information technology. The results reported in the third and fourth columns indicate that there is no significant difference between the size of upper and lower tail dependence in both periods. This result is consistent with the findings of previous papers that do not agree as to whether upper or lower tail dependence is more pronounced.

4.3. Results for extreme dependence in portfolios

In our empirical analysis so far we have focused on possible asymmetries in tail dependence using bilateral stock market data. Our results suggested that asymmetries exist when two stock markets are considered. In this section, we demonstrate the implications of asymmetrical thresholds for extreme dependence in a portfolio of stock markets⁹. To do so, we simulate different sizes of portfolios and compute our extreme dependence measure using randomly drawn asymmetric thresholds.

Before discussing the details of the methodology used, let us briefly review some underlying concepts. For each portfolio composed of d stock markets we need $d-1$ angles in order to compute our canonical conditional expectations measure (CCEM) of extreme dependence (see Eq. 9). The problem therefore becomes intractable when the angles are chosen continuously in the interval $[0, \pi/2]$ for each dimension. For this reason, we use a Monte Carlo simulation and simulate 10,000 portfolios of different sizes, from 2 stock markets to 9 stock markets in a portfolio¹⁰. Our simulations are run for the pre-EMU and the EMU period and for lower and upper tail returns separately.

In order to assess whether asymmetries are present in a portfolio of stock markets, we compare the values of our CCEM measure of extreme dependence for symmetrical thresholds and for asymmetrical thresholds. The CCEM measure of extreme dependence for symmetrical thresholds

⁹We thank an anonymous referee for this interesting suggestion.

¹⁰In the portfolio analysis we focus on EMU member countries in order to investigate possible asymmetries in extreme dependence for the EMU. However, this analysis may also be conducted for EMU and non-EMU countries in one portfolio.

is called CCEMf and is estimated using the fixed symmetrical angle $\pi/4$ in all dimensions. The CCEM measure for asymmetrical thresholds is called CCEMv and is estimated using randomly drawn angles varying in the interval $[0, \pi/2]$ for each dimension.

Using our simulation results we analyze: whether there are asymmetries in extreme dependence in portfolios of stock markets (i.e., whether there are differences between the CCEMv and the CCEMf measures); whether asymmetries in extreme dependence depend on the size of portfolios; and how the degree of extreme dependence relates to portfolio returns.

In order to answer these questions we present the results in a panel of figures summarizing the results for the simulation of 10,000 portfolios. Figure 4 presents the results for the lower tail dependence for the pre-EMU period.

The left figure in the top panel of Figure 4 displays the results for the CCEMf (i.e., the extreme dependence measure for the fixed symmetrical angle $\pi/4$) on the x-axis together with the portfolio returns on the y-axis¹¹. The mid figure in the top panel displays the results for the CCEMv (i.e., the extreme dependence measure for varying asymmetrical random angles in the interval $[0, \pi/2]$) on the x-axis together with the portfolio returns on the y-axis. In both figures, dark circles indicate portfolios with a low number of stock markets while light colors indicate large stock market portfolios. In order to assess the presence of asymmetries in extreme dependence, we display the histogram for the difference between CCEMf and CCEMv in the right figure in the top panel, where negative (positive) values indicate portfolios with asymmetrical (symmetrical) extreme dependence. In the bottom panel of Figure 4 from the left to the right we have three box plots showing the CCEMf (the left figure) and the CCEMv (the mid figure) and the difference between CCEMf and CCEMv (the right figure) for each portfolio size.

Several interesting result can be obtained from these simulation results for lower tail dependence in the pre-EMU period. First, we find that portfolios with larger values of extreme dependence experience larger stock returns. This finding is consistent with modern portfolio theory which posits that higher returns compensate investors for higher risk, in our case, for holding portfolios with a higher risk of simultaneous crashes. Second, we find that lower tail dependence

¹¹The portfolio returns are calculated using market capitalization-weighted returns of the single stock markets

increases with the number of stock markets included in a portfolio for symmetrical and asymmetrical thresholds. This finding is intuitive as the risk of simultaneous crashes rises with the number of stock markets the investor holds in her portfolio. Third, the histogram shows that a considerable number of portfolios show evidence of asymmetrical extreme dependence as indicated by the cases of negative values for the difference between $CCEM_f$ and $CCEM_v$. Thus, our simulation results largely confirm the bivariate findings of Section 4.2 that for many portfolios the risk of simultaneous crashes for asymmetrical thresholds is higher than for symmetrical thresholds. The right figure in the lower panel indicates that the presence of asymmetrical extreme dependence (as indicated by the difference between $CCEM_f$ and $CCEM_v$) varies with the portfolio size. Lower tail dependence in the pre-EMU period is particularly asymmetrical for very small portfolios (consisting of only two stock markets). This finding is also illustrated in Table 4 where we report the share of portfolios with asymmetrical lower tail dependence for different portfolios sizes¹². Again, we find evidence for asymmetrical lower tail dependence for the very small two stock market portfolios in the pre-EMU period. This finding suggests that investors can diminish the risk associated with asymmetrical tail dependence by augmenting the portfolio size.

The results for upper tail dependence for the pre-EMU period are displayed in Figure 5. Similar to the case of lower tail dependence we find that larger portfolios show higher upper tail dependence. The overall level of upper tail dependence is however lower than for lower tail dependence. That is, during the pre-EMU period stock markets are more likely to crash together than to boom together. Inspecting the histogram we find more pronounced evidence of asymmetries for upper tail dependence than for the case of lower tail dependence in the pre-EMU period. Taking asymmetrical thresholds into account is therefore even more important for assessing upper tail dependence than for assessing lower tail dependence in the pre-EMU period. The results reported in the right diagram in the lower panel of Figure 5 and in Table 4 reveal that asymmetrical upper tail dependence is a problem particularly for small portfolios. Augmenting the portfolio size may therefore diminish the risk associated with underestimating asymmetrical simultaneous booms or crashes in portfolios.

¹²Portfolios are defined to show asymmetrical tail dependence if the extreme dependence measure for an asymmetrical threshold ($CCEM_v$) is larger than for the symmetrical threshold ($CCEM_f$).

The results for lower tail dependence for the EMU period are displayed in Figure 6. Our results suggest that the correlation between the returns and the level of lower tail dependence is close to zero. That is, contrary to the pre-EMU period investors get only a negligible return premium for holding portfolios with higher extreme dependence. A possible explanation for this result may be that the increased financial integration in the EMU has also increased the occurrence of simultaneous stock market crashes leading to the fact that investors may be agnostic towards extreme dependence and may therefore not adequately consider a higher risk of lower tail dependence in stock returns.

Analogous to the results for the pre-EMU period, we find that larger portfolios have a higher risk of lower tail dependence. Compared to the results for the pre-EMU period we find a much lower level of asymmetrical extreme dependence¹³. This finding corresponds to our results for the bivariate analysis in Section 4.2. The increased symmetry of tail dependence during the EMU period may be explained by the progress in the integration of the stock markets in the EMU driven by symmetrical interest rate and exchange rate shocks for all EMU member countries, more synchronized business cycles, and an increased integration of EMU stock exchanges. Similar for the case of the pre-EMU period, the level of asymmetrical lower tail dependence is highest for portfolios consisting of two stock markets.

The results for upper tail dependence for the EMU period are displayed in Figure 7. Similar to the case of lower tail dependence during the EMU period, we find that portfolio returns and the level of upper tail dependence are only slightly related suggesting that investors can earn only very little return premiums when holding portfolios of highly dependent stock markets. Asymmetrical upper tail dependence is much lower than during the pre-EMU period which may, similar to the case of lower tail dependence, also be explained by the increased financial integration during the EMU period. Similar to the other cases of extreme dependence, the level of asymmetries in upper tail dependence in the EMU period is highest for the smallest portfolios (consisting of two stocks) suggesting that investors can reduce the risk of asymmetrical tail dependence by holding large portfolios. The level of upper tail dependence is lower than the level of lower tail dependence for

¹³For example, the histogram shows that the difference between the CCEMf and the CCEMv measure is mostly positive, i.e., 82% of the portfolios of size three to nine have their largest value at the symmetrical threshold ($\pi/4$).

the EMU period which suggest that, similar to the pre-EMU period, simultaneous crashes occur more frequently than simultaneous booms.

Overall, the portfolio simulation results reveal several interesting insights into the nature of extreme dependence of stock markets in the EMU. While asymmetries in extreme dependence are present for all four cases studied, they are more pronounced for the pre-EMU period and for upper tail dependence than for the EMU period and for lower tail dependence. In general, the level of asymmetries in extreme dependence is larger for small portfolios than for large portfolios suggesting that the risk associated with asymmetrical tail dependence may be diminished by augmenting portfolios. That is, investors should be aware that the probability of simultaneous stock markets crashes and simultaneous booms can be higher for asymmetrical crash or boom thresholds - but when holding portfolios consisting of a large number of stock markets the risk of underestimating tail dependence may be reduced. However, portfolio diversification has limits since larger portfolios have higher levels of extreme stock market comovement. Moreover, we find that higher levels of tail dependence are associated with higher portfolio returns in the pre-EMU period while no such correlation is found for the EMU period.

5. Conclusions

The recent decades have shown that stock markets tend to crash and boom together. In this paper, we investigate extreme dependence using asymmetrical thresholds of crash or boom quantiles. Using daily data for ten initial EMU member countries, the United Kingdom, and the United States in the pre-EMU period (1989-1998) and the EMU period (1999-2010), we find that co-crashes and co-booms are largely asymmetrical in the pre-EMU period and largely symmetrical in the EMU period. Quantifying extreme dependence solely for the special case of symmetrical thresholds may therefore lead to an underestimation of the probability of extreme stock markets events. In order to create efficient stock portfolios and to achieve the maximum amount of diversification benefits, investors should thus also consider the possibility of simultaneous crashes or booms with asymmetric thresholds.

Our results indicate that after the formation of the EMU co-booms and co-crashes have become more symmetrical among the EMU member countries. This increased symmetry in extreme

stock market dependence may be explained by the common monetary policy conducted by the European Central Bank (which has led to symmetrical interest rate and exchange rate shocks in the EMU), more synchronized business cycles, and a higher integration of European stock markets. Overall, the degree of extreme dependence has increased among the stock markets of the EMU member countries after the introduction of the euro, which reflects the increased financial and economic integration triggered by more symmetrical business cycles, larger cross-border capital flows, a reduction of financial regulation, new electronic trading systems, and innovations in information technology. We do not find a significant difference between the size of upper and lower tail dependence which resembles the mixed findings with regard to this issue in the literature. We find that extreme dependence of the EMU members' stock markets with respect to the stock markets of the United Kingdom and the United States has increased following the introduction of the euro, whereby this effect is more pronounced for the United Kingdom suggesting that the UK stock market has become much more integrated into the European stock markets than the US stock market.

Our results suggest that portfolios with larger values of extreme dependence experience larger stock returns, at least for the pre-EMU period. This result is consistent with modern portfolio theory and suggests that investors demand higher returns as a compensation for the higher risk of extreme dependence of stock markets. Our portfolio analysis reveals that the degree of asymmetries is lower for larger portfolios, which suggests that investors can diminish the risk associated with asymmetrical tail dependence by augmenting the portfolio size. However, as suggested by our results, increasing the the portfolio size may lead to higher risk of extreme stock market co-movement, which clearly shows the limits of portfolio diversification.

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Proofs

Proof. Proposition 2 □

1. For the demonstration of the Property 1 see appendix in Hsing et al. (2004).
2. Note that by definition

$$\mathbb{P}\left(\bigcup_{j=1}^d X_j > x_j\right) \sim \frac{1}{n} \Lambda\left(n\bar{F}(x_1), \dots, n\bar{F}(x_d)\right) \quad (.1)$$

for $n \rightarrow \infty$. Then, replacing $t = n/\alpha$ in (.1) such that $t \rightarrow \infty$. We have

$$\begin{aligned} \mathbb{P}\left(\bigcup_{j=1}^d X_j > x_j\right) &\sim \frac{1}{t\alpha} \Lambda\left(t\alpha\bar{F}(x_1), \dots, t\alpha\bar{F}(x_d)\right) \\ &= \alpha^{-1}\bar{F}(x_1) \Lambda\left(\alpha, \alpha\frac{\bar{F}(x_2)}{\bar{F}(x_1)}, \dots, \alpha\frac{\bar{F}(x_d)}{\bar{F}(x_1)}\right). \end{aligned} \quad (.2)$$

On the other hand we have

$$\psi(\varphi_1, \dots, \varphi_{d-1}) := \frac{\mathbb{P}\left(\bigcup_{j=1}^d X_j > x_j\right)}{\mathbb{P}(X_1 > x_1)}$$

Now dividing by $\bar{F}(x_1)$ in (.2) we obtain the wished result.

3. Let be a_0, \dots, a_{d-1} a series of positive constants and for ease of the notation $\tan \varphi_0 = 1$ for all $\varphi_0 \in [0, \pi/2]$. Furthermore, let $a_{\min} := 1 \wedge \bigwedge_{j=1}^{d-1} a_j$, then for Definition 1 and the homogeneity feature in Proposition 2 we obtain

$$\begin{aligned} \psi(a_1\varphi_1, \dots, a_{d-1}\varphi_{d-1}) &= \lim_{t \rightarrow \infty} t\mathbb{P}\left(\bigcup_{k=0}^{k=d-1} t\bar{F}_{k+1}(X_{1,k+1}) \leq a_k \tan \varphi_k\right) \\ &= \lim_{t \rightarrow \infty} a_{\min} t\mathbb{P}\left(\bigcup_{k=0}^{k=d-1} t\bar{F}_{k+1}(X_{1,k+1}) \leq \frac{a_k}{a_{\min}} \tan \varphi_k\right) \\ &= a_{\min} \psi\left(\frac{a_1}{a_{\min}}\varphi_1, \dots, \frac{a_{d-1}}{a_{\min}}\varphi_{d-1}\right) \geq a_{\min} \psi(\varphi_1, \dots, \varphi_{d-1}). \end{aligned}$$

The other bound can be demonstrated similarly.

4. For the demonstration of the Property 4 see Corollary in Brodin and Klueppelberg (2010).

Figures and Tables

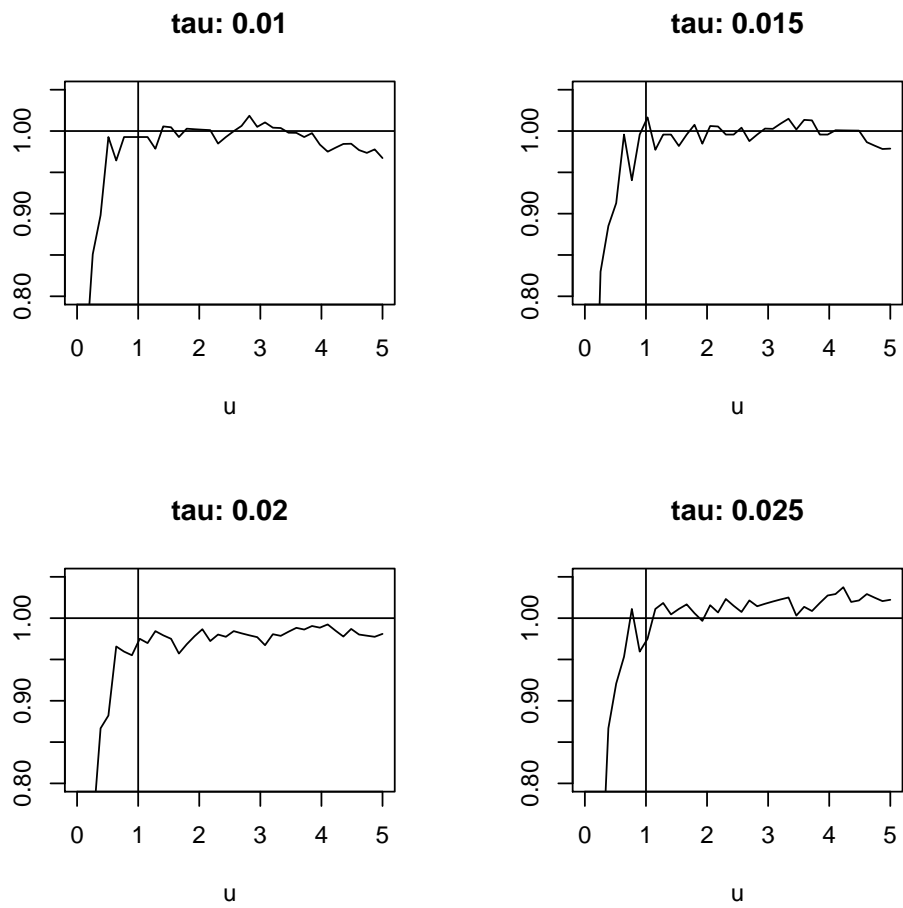


Figure 1: Example of the stock market pair Germany and France for the pre-EMU period for the choice of the parameter τ .

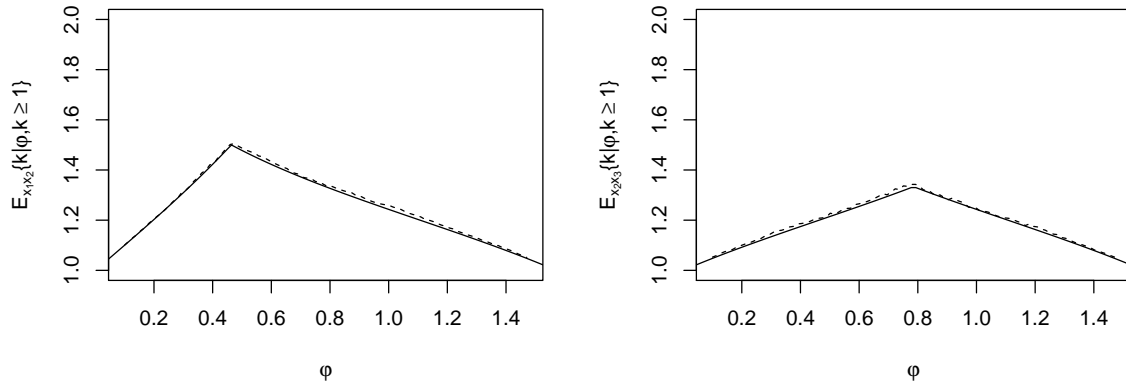


Figure 2: Empirical (the dashed line) and theoretical (the solid line) simulation of the measure of extreme dependence $E\{k|\varphi, k \geq 1\}$ for the pairs (X_1, X_2) (left) and (X_2, X_3) (right) in the Example 3.1.

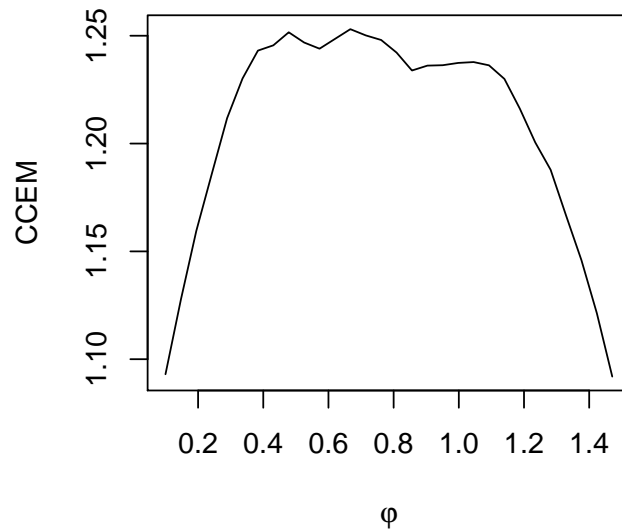


Figure 3: CCEM of the stock market pair Germany and France for the pre-EMU period for the continuous range of $\varphi \in [0, \pi/2]$.

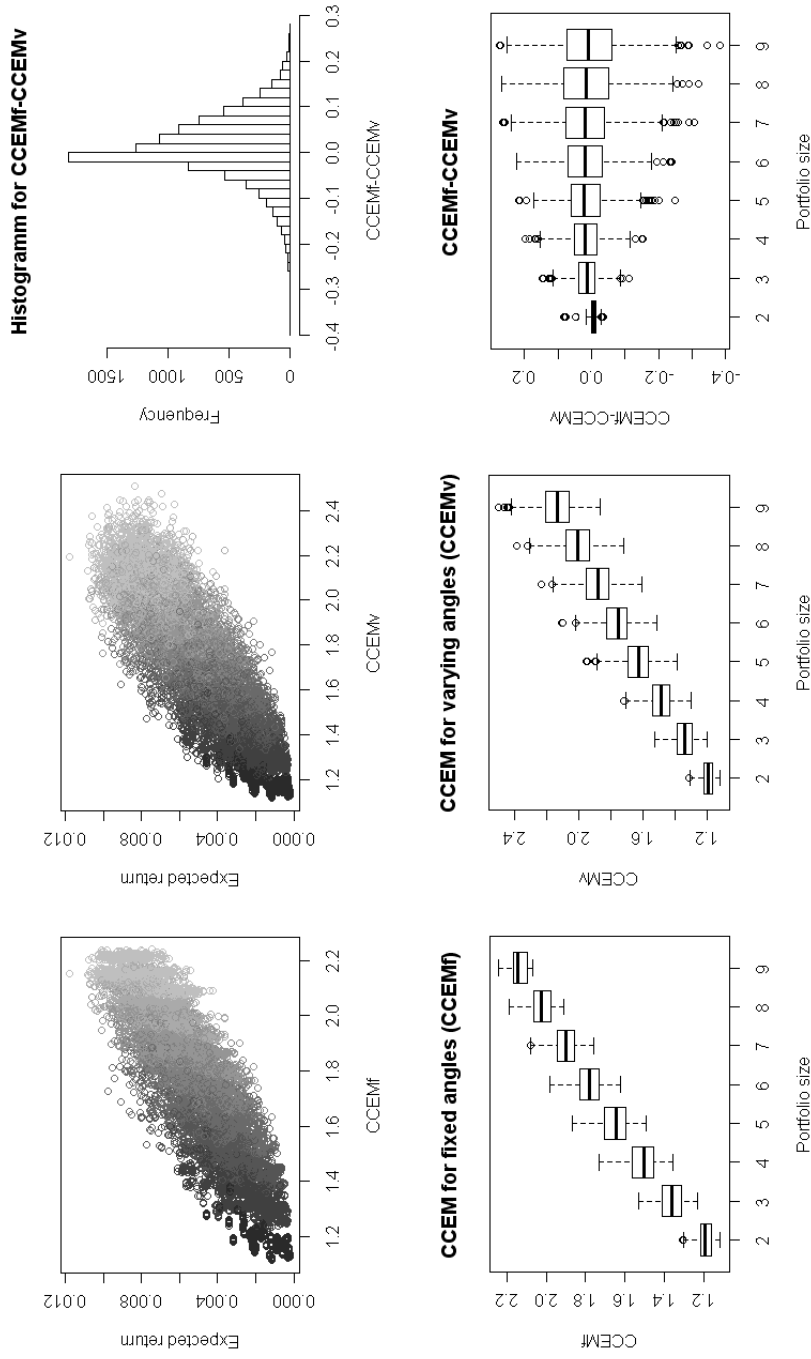


Figure 4: Simulation results for lower tail dependence for the pre-EMU period. The left figure in the top panel displays the results for the CCEMF (extreme dependence for the symmetrical threshold) on the x-axis together with the portfolio returns in the y-axis. The mid figure in the top panel displays the results for the CCEMv (extreme dependence for an asymmetrical threshold) on the x-axis together with the portfolio returns on the y-axis. In both figures, dark circles indicate portfolios with a low number of stock markets while light colors indicate large stock market portfolios. In addition, we display the histogram for the difference between CCEMf and CCEMv in the right figure in the top panel, where negative (positive) values indicate portfolios with asymmetrical (symmetrical) extreme dependence. In the bottom panel from the left to the right we have three box plots showing the CCEMf (the left figure) and the CCEMv (the mid figure) and the difference between CCEMf and CCEMv for each portfolio size.

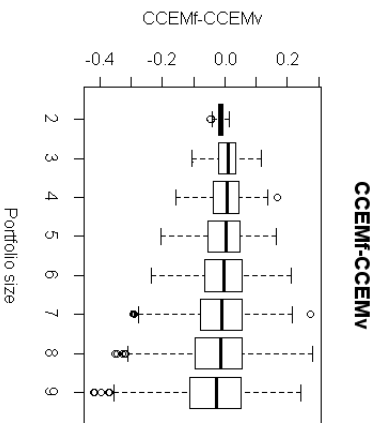
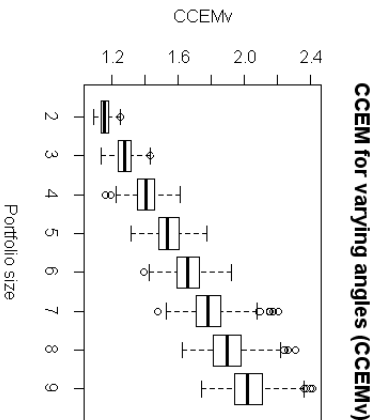
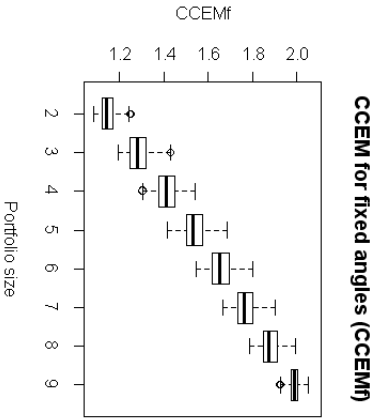
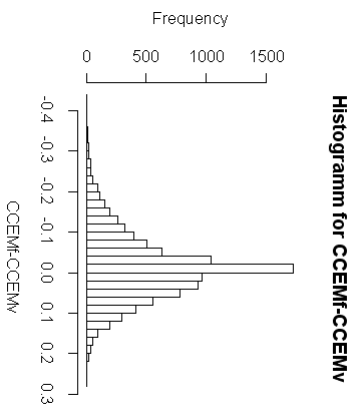
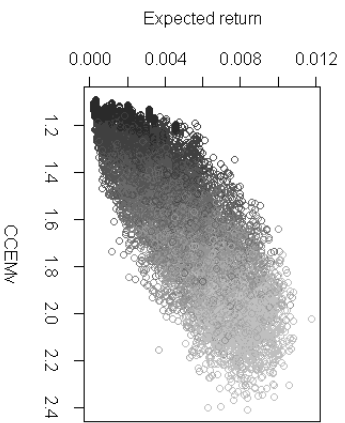
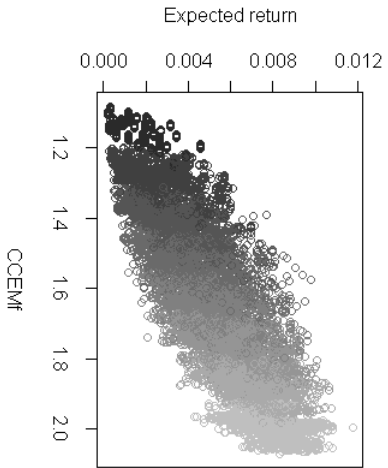
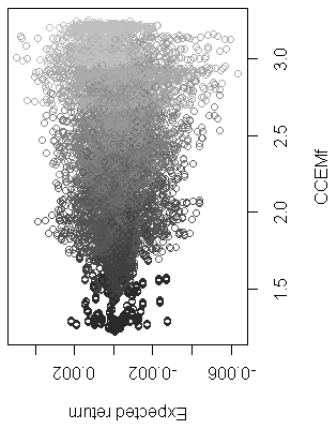
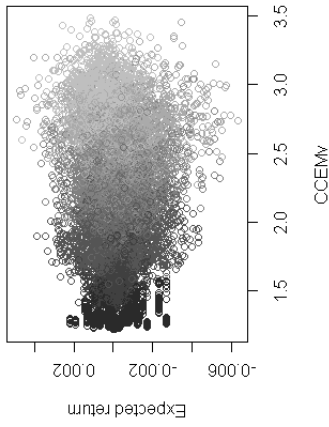
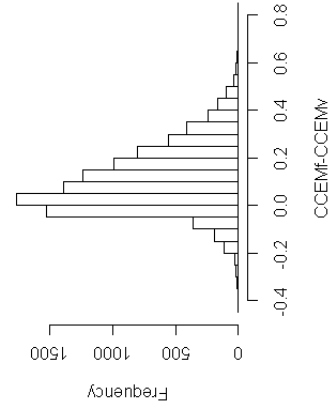
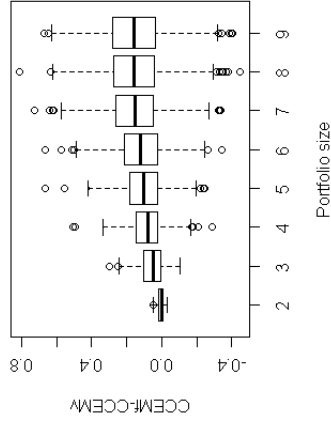


Figure 5: Simulation results for upper tail dependence for the pre-EMU period. The left figure in the top panel displays the results for the CCEMF (extreme dependence for the symmetrical threshold) on the x-axis together with the portfolio returns in the y-axis. The mid figure in the top panel displays the results for the CCEMV (extreme dependence for an asymmetrical threshold) on the x-axis together with the portfolio returns on the y-axis. In both figures, dark circles indicate portfolios with a low number of stock markets while light colors indicate large stock market portfolios. In addition, we display the histogram for the difference between CCEMF and CCEMV in the right figure in the top panel, where negative (positive) values indicate portfolios with asymmetrical (symmetrical) extreme dependence. In the bottom panel from the left to the right we have three box plots showing the CCEMF (the left figure) and the CCEMV (the mid figure) and the difference between CCEMF and CCEMV for each portfolio size.

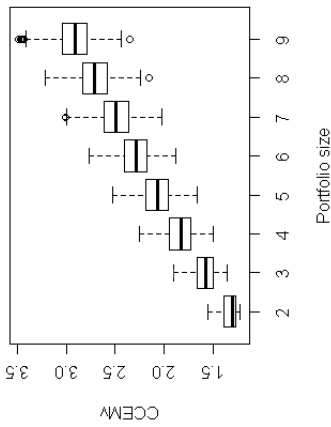
Histogramm for CCEMF-CCEMv



CCEMF-CCEMv



CCEM for varying angles (CCEMv)



CCEM for fixed angles (CCEMf)

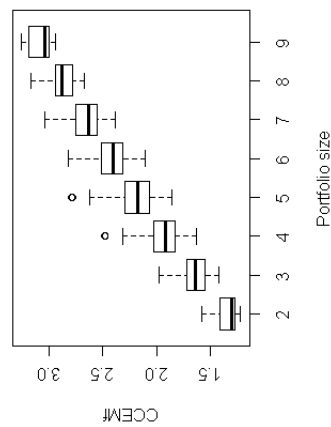


Figure 6: Simulation results for lower tail dependence for the EMU period. The left figure in the top panel displays the results for the CCEMF (extreme dependence for the symmetrical threshold) on the x-axis together with the portfolio returns in the y-axis. The mid figure in the top panel displays the results for the CCEMv (extreme dependence for an asymmetrical threshold) on the x-axis together with the portfolio returns on the y-axis. In both figures, dark circles indicate portfolios with a low number of stock markets while light colors indicate large stock market portfolios. In addition, we display the histogram for the difference between CCEMf and CCEMv in the right figure in the top panel, where negative (positive) values indicate portfolios with asymmetrical (symmetrical) extreme dependence. In the bottom panel from the left to the right we have three box plots showing the CCEMf (the left figure) and the CCEMv (the mid figure) and the difference between CCEMf and CCEMv for each portfolio size.

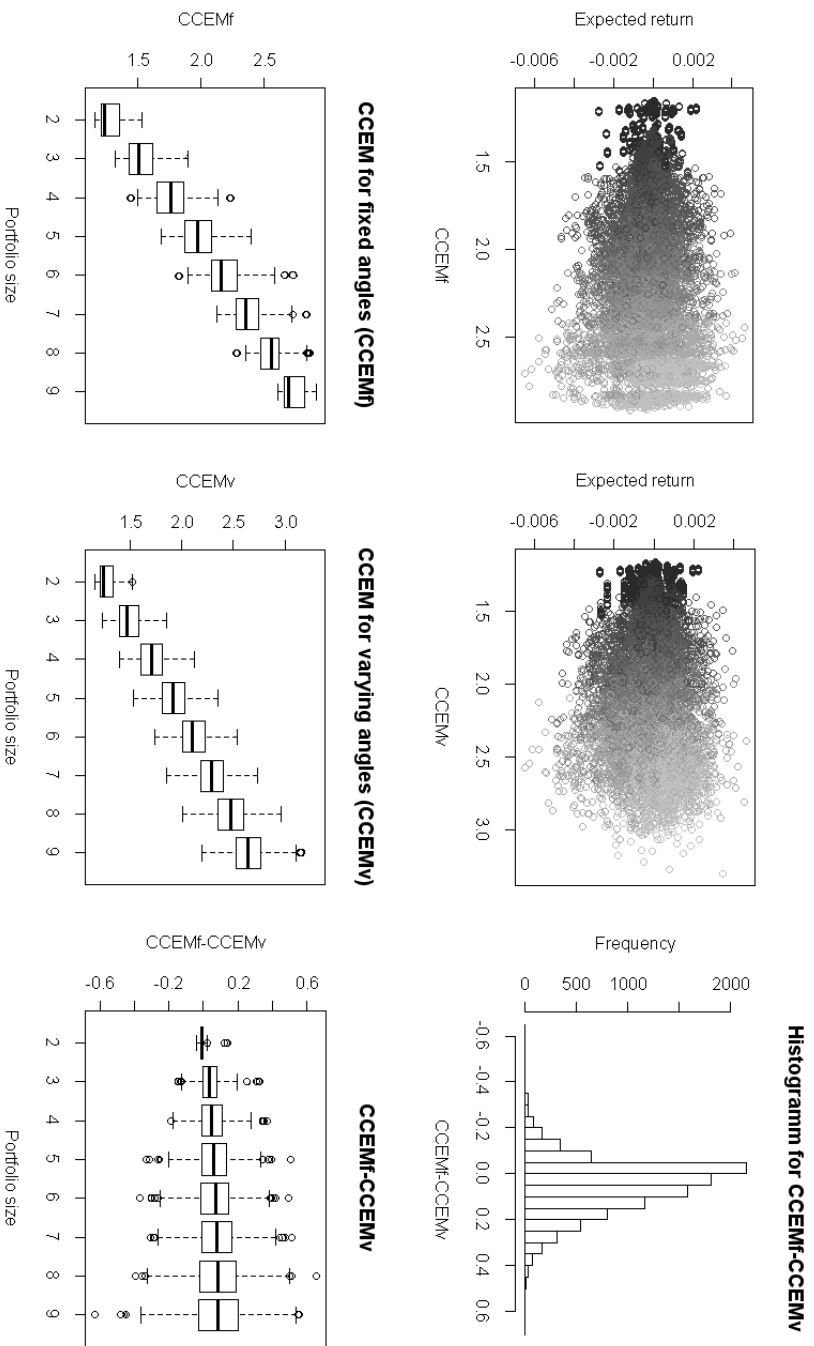


Figure 7: Simulation results for upper tail dependence for the EMU period. The left figure in the top panel displays the results for the CCEMF (extreme dependence for the symmetrical threshold) on the x-axis together with the portfolio returns in the y-axis. The mid figure in the top panel displays the results for the CCEMV (extreme dependence for an asymmetrical threshold) on the x-axis together with the portfolio returns on the y-axis. In both figures, dark circles indicate portfolios with a low number of stock markets while light colors indicate large stock market portfolios. In addition, we display the histogram for the difference between CCEMF and CCEMV in the right figure in the top panel, where negative (positive) values indicate portfolios with asymmetrical (symmetrical) extreme dependence. In the bottom panel from the left to the right we have three box plots showing the CCEMF (the left figure) and the CCEMV (the mid figure) and the difference between CCEMF and CCEMV for each portfolio size.

Stocks	mean	sd	min	max	skewness	kurtosis	Box.test	Jarque-Bera	ADW	Engle (10)
AT	0.028	1.305	-9.337	7.631	-0.334	6.933	*124.372	*5284.550	*-11.794	*276.9764
BE	0.038	0.718	-5.313	6.654	-0.336	9.763	*106.240	*10432.030	*-11.098	*139.5082
FI	0.043	1.249	-8.582	9.686	-0.124	5.663	*98.324	*3500.677	*-11.393	*446.1813
FR	0.035	1.167	-7.601	6.830	-0.212	3.282	*13.033	*1194.104	*-13.092	*204.8791
DE	0.051	1.224	-13.696	7.308	-0.975	11.559	*9.415	*14967.550	*-12.860	*138.1616
IE	0.049	0.887	-7.555	6.028	-0.401	9.048	96.197	*8988.625	*-11.863	*199.4565
NL	0.058	1.035	-6.781	5.724	-0.399	5.020	*8.922	*2815.345	*-11.349	*405.1695
PT	0.046	0.855	-10.802	7.591	-1.188	25.096	*179.015	*69199.919	*-10.955	*125.8954
ES	0.044	1.090	-8.641	6.381	-0.528	6.372	*29.993	*4545.147	*-11.378	*233.0527
UK	0.045	0.865	-4.141	5.438	0.057	2.504	*22.462	*685.356	*-13.090	*230.6357
US	0.057	0.841	-7.113	4.989	-0.569	7.521	**4.678	*6303.021	*-13.422	*198.1214
IT	0.041	1.414	-8.416	6.974	-0.166	2.625	*17.826	*763.785	*-12.585	*256.0747

Table 1: Descriptive statistics for the stock markets returns denominated in US dollars for the pre-EMU period. This table shows the summary statistics for the stock index returns for the pre-EMU period (1989 - 1998). *, **, *** denote statistical significance at the 1, 5 and 10 % level respectively. The Ljung-Box test statistic tests for serial dependence up to the 5-th order.

Stocks	mean	sd	min	max	skewness	kurtosis	Box.test	Jarque-Bera	ADW	Engle (10)
AT	0.026	1.449	-10.264	12.010	-0.333	9.068	*13.266	*10349.660	*-13.143	*866.6119
BE	-0.006	1.208	-8.656	10.324	0.004	7.742	*32.729	*7504.245	*-13.647	*729.1681
FI	0.004	2.072	-17.404	14.550	-0.328	6.137	*6.750	*4769.741	*-13.749	*141.1148
FR	-0.007	1.544	-9.471	10.580	0.041	5.122	*40.162	*3285.971	*-14.139	*545.3122
DE	0.007	1.616	-8.874	10.770	0.032	4.350	*16.282	*2370.338	*-14.280	*546.8545
IE	-0.018	1.478	-13.946	9.728	-0.596	8.002	*9.652	*8194.270	*-14.572	*750.5512
NL	-0.016	1.584	-9.585	10.024	-0.065	5.974	*38.294	*4471.024	*-13.724	*778.9085
PT	-0.001	1.089	-10.649	10.108	-0.130	12.084	*17.860	*18286.279	*-12.194	*424.5737
ES	0.003	1.395	-9.675	13.733	0.136	7.852	*17.025	*7727.306	*-14.476	*433.2817
UK	-0.005	1.300	-9.265	9.386	-0.134	6.103	*67.063	*4672.827	*-14.957	*695.6186
US	-0.006	1.350	-9.470	10.957	-0.105	7.606	*35.482	*7245.582	-14.375	*780.0473
IT	-0.017	1.357	-8.132	9.872	0.005	5.410	*38.648	*3664.249	*-13.563	*542.5238

Table 1 (continued): Descriptive statistics for the stock markets returns denominated in US dollars for the EMU period. This table shows the summary statistics for the stock index return for the EMU period (1999 - 2010). *, **, *** denote statistical significance at the 1, 5 and 10 % level respectively. The Ljung-Box test statistic tests for serial dependence up to the 5-th order.

Countries	Lower Tail						Upper Tail					
	Sample 1989-1998			Sample 1999-2007			Sample 1989-1998			Sample 1999-2007		
	$\hat{E}_\tau^L \{k \mid \varphi, k \geq 1\}$			$\hat{E}_\tau^L \{k \mid \varphi, k \geq 1\}$			$\hat{E}_\tau^U \{k \mid \varphi, k \geq 1\}$			$\hat{E}_\tau^U \{k \mid \varphi, k \geq 1\}$		
	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$
AT - BE	1.173	1.189	1.199	1.259	1.259	1.232	1.170	1.136	1.163	1.191	1.185	1.199
AT - FI	1.168	1.158	1.160	1.254	1.253	1.236	1.136	1.101	1.138	1.214	1.195	1.195
AT - FR	1.185	1.209	1.202	1.260	1.293	1.254	1.163	1.116	1.148	1.207	1.203	1.207
AT - DE	1.229	1.246	1.255	1.251	1.277	1.259	1.210	1.189	1.217	1.206	1.210	1.206
AT - IE	1.182	1.190	1.177	1.223	1.242	1.221	1.148	1.142	1.149	1.184	1.167	1.195
AT - NL	1.216	1.202	1.192	1.264	1.302	1.255	1.195	1.197	1.199	1.216	1.217	1.223
AT - PT	1.148	1.134	1.149	1.229	1.242	1.228	1.122	1.102	1.133	1.192	1.161	1.181
AT - ES	1.186	1.172	1.193	1.251	1.290	1.252	1.154	1.137	1.155	1.200	1.187	1.206
AT - UK	1.172	1.177	1.161	1.271	1.289	1.253	1.142	1.119	1.141	1.205	1.184	1.194
AT - US	1.125	1.113	1.124	1.153	1.140	1.156	1.096	1.075	1.098	1.154	1.131	1.139
AT - IT	1.171	1.167	1.184	1.252	1.281	1.232	1.142	1.122	1.149	1.189	1.166	1.182
BE - FI	1.168	1.140	1.154	1.268	1.303	1.269	1.173	1.148	1.176	1.245	1.248	1.237
BE - FR	1.192	1.201	1.183	1.321	1.418	1.325	1.168	1.160	1.158	1.288	1.346	1.299
BE - DE	1.222	1.248	1.223	1.308	1.357	1.308	1.200	1.190	1.202	1.287	1.328	1.291
BE - IE	1.200	1.173	1.172	1.229	1.247	1.245	1.170	1.163	1.163	1.216	1.209	1.226
BE - NL	1.223	1.215	1.211	1.322	1.418	1.322	1.199	1.205	1.197	1.297	1.356	1.320
BE - PT	1.165	1.160	1.158	1.251	1.287	1.249	1.147	1.154	1.171	1.205	1.182	1.194
BE - ES	1.186	1.196	1.178	1.298	1.343	1.297	1.164	1.167	1.170	1.269	1.323	1.285
BE - UK	1.176	1.160	1.161	1.295	1.380	1.301	1.156	1.130	1.151	1.275	1.262	1.262
BE - US	1.112	1.105	1.114	1.184	1.189	1.189	1.113	1.095	1.121	1.198	1.181	1.193
BE - IT	1.178	1.169	1.183	1.310	1.370	1.300	1.160	1.141	1.154	1.268	1.302	1.271
FI - FR	1.153	1.152	1.167	1.321	1.402	1.329	1.144	1.121	1.152	1.285	1.359	1.313
FI - DE	1.175	1.191	1.192	1.298	1.365	1.312	1.154	1.132	1.165	1.274	1.324	1.289
FI - IE	1.149	1.160	1.157	1.241	1.263	1.240	1.137	1.119	1.142	1.223	1.219	1.219
FI - NL	1.164	1.155	1.161	1.325	1.394	1.331	1.167	1.153	1.183	1.285	1.357	1.317
FI - PT	1.139	1.133	1.152	1.240	1.278	1.244	1.133	1.121	1.141	1.225	1.197	1.216
FI - ES	1.159	1.159	1.166	1.280	1.336	1.286	1.149	1.136	1.160	1.264	1.309	1.290
FI - UK	1.157	1.173	1.175	1.310	1.356	1.286	1.123	1.117	1.150	1.274	1.308	1.285
FI - US	1.114	1.108	1.115	1.184	1.194	1.203	1.089	1.074	1.107	1.175	1.168	1.186
FI - IT	1.153	1.136	1.150	1.301	1.353	1.297	1.131	1.106	1.147	1.272	1.290	1.266
FR - DE	1.263	1.281	1.243	1.396	1.563	1.394	1.202	1.191	1.214	1.372	1.502	1.359
FR - IE	1.206	1.214	1.192	1.253	1.294	1.268	1.165	1.141	1.164	1.226	1.211	1.236
FR - NL	1.301	1.373	1.290	1.397	1.573	1.396	1.245	1.245	1.261	1.375	1.530	1.379

Table 2: Result of the extreme dependence measures $E \{k \mid \varphi, k \geq 1\}$ for the country pairs of ten initial EMU member countries, the United States, and the United Kingdom. The returns are denominated in US dollars.

Countries	Lower Tail						Upper Tail					
	Sample 1989-1998			Sample 1999-2007			Sample 1989-1998			Sample 1999-2007		
	$\hat{E}_\tau^L \{k \mid \varphi, k \geq 1\}$			$\hat{E}_\tau^L \{k \mid \varphi, k \geq 1\}$			$\hat{E}_\tau^U \{k \mid \varphi, k \geq 1\}$			$\hat{E}_\tau^U \{k \mid \varphi, k \geq 1\}$		
	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$
FR - PT	1.148	1.157	1.143	1.286	1.315	1.267	1.125	1.113	1.128	1.250	1.226	1.229
FR - ES	1.263	1.299	1.245	1.375	1.520	1.369	1.210	1.195	1.222	1.350	1.437	1.349
FR - UK	1.275	1.310	1.267	1.381	1.527	1.364	1.231	1.224	1.231	1.344	1.449	1.355
FR - US	1.169	1.167	1.158	1.221	1.241	1.235	1.154	1.133	1.153	1.223	1.230	1.225
FR - IT	1.220	1.239	1.214	1.383	1.554	1.374	1.177	1.150	1.164	1.352	1.457	1.351
DE - IE	1.226	1.198	1.188	1.237	1.272	1.254	1.151	1.129	1.173	1.218	1.205	1.226
DE - NL	1.272	1.310	1.271	1.383	1.539	1.382	1.242	1.218	1.247	1.338	1.447	1.347
DE - PT	1.164	1.165	1.158	1.268	1.295	1.263	1.145	1.138	1.151	1.245	1.216	1.217
DE - ES	1.216	1.257	1.221	1.356	1.439	1.361	1.173	1.166	1.187	1.340	1.397	1.332
DE - UK	1.229	1.220	1.216	1.327	1.442	1.350	1.176	1.149	1.166	1.315	1.353	1.324
DE - US	1.129	1.114	1.142	1.251	1.269	1.254	1.109	1.092	1.122	1.244	1.264	1.242
DE - IT	1.191	1.187	1.180	1.370	1.481	1.363	1.152	1.130	1.163	1.339	1.404	1.338
IE - NL	1.208	1.222	1.215	1.273	1.297	1.256	1.179	1.176	1.179	1.232	1.227	1.235
IE - PT	1.142	1.133	1.148	1.226	1.222	1.208	1.107	1.090	1.117	1.200	1.175	1.184
IE - ES	1.176	1.190	1.191	1.258	1.305	1.259	1.141	1.132	1.147	1.212	1.207	1.219
IE - UK	1.226	1.230	1.212	1.283	1.303	1.266	1.189	1.165	1.186	1.253	1.225	1.238
IE - US	1.125	1.109	1.141	1.165	1.164	1.174	1.111	1.095	1.109	1.155	1.125	1.147
IE - IT	1.163	1.174	1.168	1.254	1.261	1.240	1.143	1.117	1.141	1.212	1.199	1.206
NL - PT	1.158	1.160	1.148	1.276	1.318	1.276	1.142	1.134	1.141	1.245	1.228	1.214
NL - ES	1.247	1.272	1.238	1.357	1.456	1.345	1.207	1.213	1.210	1.322	1.394	1.326
NL - UK	1.291	1.331	1.285	1.346	1.475	1.356	1.244	1.201	1.220	1.337	1.409	1.336
NL - US	1.164	1.169	1.157	1.225	1.243	1.224	1.132	1.118	1.152	1.213	1.230	1.216
NL - IT	1.206	1.212	1.203	1.373	1.483	1.350	1.193	1.177	1.172	1.338	1.425	1.342
PT - ES	1.158	1.159	1.147	1.274	1.317	1.305	1.143	1.123	1.155	1.244	1.247	1.261
PT - UK	1.146	1.119	1.132	1.269	1.324	1.284	1.125	1.101	1.121	1.215	1.215	1.232
PT - US	1.116	1.106	1.106	1.171	1.174	1.189	1.094	1.075	1.093	1.157	1.144	1.160
PT - IT	1.137	1.123	1.135	1.263	1.293	1.266	1.129	1.098	1.132	1.218	1.219	1.226
ES - UK	1.233	1.241	1.214	1.329	1.402	1.333	1.202	1.184	1.194	1.323	1.371	1.307
ES - US	1.130	1.124	1.129	1.210	1.220	1.217	1.135	1.131	1.134	1.205	1.213	1.204
ES - IT	1.199	1.216	1.194	1.354	1.480	1.350	1.205	1.161	1.166	1.339	1.397	1.325
UK - US	1.173	1.162	1.174	1.221	1.227	1.221	1.152	1.138	1.154	1.216	1.229	1.216
UK - IT	1.195	1.182	1.175	1.351	1.427	1.329	1.159	1.151	1.170	1.304	1.362	1.321
US - IT	1.128	1.118	1.127	1.226	1.241	1.221	1.114	1.087	1.102	1.211	1.234	1.224

Table 2 (continued): Result of the extreme dependence measures $E \{k \mid \varphi, k \geq 1\}$ for the country pairs of ten initial EMU member countries, the United States, and the United Kingdom. The returns are denominated in US dollars.

Countries	Lower tail first and second period			Upper tail first and second period			Upper and lower tail first period			Upper and lower tail second period		
	$H_0 : \hat{E}_{\tau,1}^L = \hat{E}_{\tau,2}^L$			$H_0 : \hat{E}_{\tau,1}^U = \hat{E}_{\tau,2}^U$			$H_0 : \hat{E}_{\tau,1}^L = \hat{E}_{\tau,1}^U$			$H_0 : \hat{E}_{\tau,2}^L = \hat{E}_{\tau,2}^U$		
	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$
AT - BE	-2.06	-1.73	-1.66	-1.24	-1.00	-0.72	0.44	0.69	0.05	2.35	1.49	1.02
AT - FI	-1.95	-1.79	-1.68	-2.59	-2.73	-4.02	0.37	0.94	1.44	0.47	0.86	0.89
AT - FR	-1.95	-1.65	-1.85	-1.69	-3.28	-2.86	0.59	2.23	1.95	1.57	2.10	1.99
AT - DE	-0.94	-0.92	-0.88	0.04	-0.00	1.15	0.45	1.52	1.33	0.24	1.63	1.91
AT - IE	-1.27	-1.34	-1.59	-1.73	-0.55	-1.68	0.46	0.66	0.96	0.18	1.70	1.72
AT - NL	-1.41	-1.83	-1.98	-0.72	-0.95	-0.06	0.24	0.04	-0.39	0.90	1.68	1.84
AT - PT	-2.35	-2.05	-2.06	-2.94	-2.55	-3.46	0.22	0.95	1.19	0.67	1.03	1.26
AT - ES	-1.73	-1.81	-1.90	-1.17	-0.92	-1.35	0.16	0.50	0.79	1.71	2.08	1.51
AT - UK	-1.87	-1.74	-2.04	-1.40	-1.75	-3.53	0.28	1.09	1.16	2.04	1.68	1.63
AT - US	-3.16	-1.53	-1.83	-2.53	-3.02	-1.80	0.64	1.25	0.88	0.03	0.11	0.63
AT - IT	-2.03	-1.53	-1.74	-5.63	-1.88	-1.65	2.14	1.38	1.03	1.58	2.36	2.18
BE - FI	-2.54	-2.93	-2.87	-3.04	-4.28	-8.37	-0.24	-0.07	-0.81	0.63	0.78	0.45
BE - FR	-3.17	-3.25	-3.36	-5.82	-7.13	-6.49	0.53	0.52	1.04	2.01	1.05	0.27
BE - DE	-2.04	-2.20	-2.41	-3.54	-5.49	-7.42	0.25	1.57	0.14	0.66	0.24	0.65
BE - IE	-2.58	-2.50	-2.27	-2.82	-1.78	-2.36	0.08	0.13	0.30	0.82	1.10	0.82
BE - NL	-3.16	-3.00	-3.28	-6.87	-5.14	-4.76	0.51	0.62	0.07	1.44	0.90	0.29
BE - PT	-2.99	-2.51	-2.62	-2.66	-1.19	-2.51	0.32	0.58	-0.18	1.88	2.64	1.34
BE - ES	-3.65	-3.43	-3.44	-6.13	-6.96	-5.92	0.76	0.54	0.31	1.37	1.17	0.43
BE - UK	-3.24	-3.30	-3.46	-5.57	-6.97	-8.66	0.37	1.20	0.99	1.89	1.90	0.74
BE - US	-3.51	-2.01	-2.43	-4.54	-3.82	-2.37	-0.28	0.51	-0.79	0.14	0.11	0.09
BE - IT	-4.19	-3.60	-3.34	-8.29	-10.11	-11.22	0.01	1.98	1.27	2.22	1.57	0.51
FI - FR	-3.93	-4.33	-5.10	-6.03	-5.98	-8.41	0.70	0.51	0.60	0.63	0.98	0.53
FI - DE	-2.84	-2.90	-3.63	-3.89	-3.04	-5.80	1.06	0.63	0.27	0.85	0.98	1.16
FI - IE	-2.54	-2.05	-2.46	-2.17	-2.23	-2.79	0.33	0.51	0.33	0.92	0.46	0.68
FI - NL	-3.16	-3.87	-3.65	-9.38	-4.75	-12.91	-1.64	0.06	-0.67	0.72	0.55	0.02
FI - PT	-3.34	-3.08	-2.51	-9.21	-4.59	-6.13	0.06	0.71	0.91	0.13	0.61	0.28
FI - ES	-2.70	-3.17	-3.31	-5.46	-5.13	-5.45	1.23	1.01	0.03	0.40	0.20	0.24
FI - UK	-3.13	-3.26	-3.19	-4.44	-5.47	-9.01	1.00	1.25	1.29	0.78	0.27	0.10
FI - US	-3.35	-2.73	-2.74	-4.61	-6.17	-7.99	0.80	1.46	1.33	0.04	0.02	0.14
FI - IT	-4.15	-3.80	-4.46	-3.63	-3.79	-6.40	0.51	0.69	0.56	1.24	1.54	0.96
FR - DE	-6.21	-4.31	-6.04	-6.29	-9.79	-5.89	1.13	1.90	0.57	0.73	1.26	0.50
FR - IE	-2.95	-2.45	-2.80	-3.51	-2.22	-4.51	1.24	1.49	2.15	2.20	2.26	0.96
FR - NL	-7.22	-4.99	-5.19	-5.47	-6.86	-4.12	1.29	1.70	0.62	1.10	1.45	1.00

Table 3: The t-tests for equality in extreme dependence for the stock market returns in US dollars. The first and second columns report the result of the t-values testing whether lower and upper tail dependence change significantly after the introduction of the euro. Values below -1.96 (above 1.96) indicate that extreme dependence becomes significantly larger (smaller) after the formation of the EMU at the 5% level of significance. The third and fourth columns report the t-values testing the null that upper and lower tail dependence are equal in the pre-EMU or the EMU period. Values below -1.96 indicate that upper tail dependence is significantly more pronounced than lower tail dependence and values above 1.96 indicate that lower tail dependence is significantly more pronounced than upper tail dependence in the respective period.

Countries	Lower tail first and second period			Upper tail first and second period			Upper and lower tail first period			Upper and lower tail second period		
	$H_0 : \hat{E}_{\tau,1}^L = \hat{E}_{\tau,2}^L$			$H_0 : \hat{E}_{\tau,1}^U = \hat{E}_{\tau,2}^U$			$H_0 : \hat{E}_{\tau,1}^L = \hat{E}_{\tau,1}^U$			$H_0 : \hat{E}_{\tau,2}^L = \hat{E}_{\tau,2}^U$		
	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/8$	$\pi/4$	$3\pi/8$
FR - PT	-3.80	-2.98	-2.98	-5.70	-3.59	-8.75	0.43	0.76	1.89	0.44	1.12	0.53
FR - ES	-4.01	-5.05	-5.49	-5.20	-7.72	-4.65	0.19	1.80	0.25	0.03	0.69	0.48
FR - UK	-4.69	-2.77	-4.18	-3.90	-8.00	-5.81	0.46	2.48	1.01	1.28	0.38	0.20
FR - US	-2.66	-1.91	-2.46	-3.19	-2.67	-1.86	0.56	0.62	0.24	0.08	0.16	0.21
FR - IT	-5.04	-4.85	-4.97	-5.97	-16.02	-10.09	0.74	3.45	1.46	1.82	1.47	0.22
DE - IE	-2.23	-1.76	-2.11	-2.88	-1.07	-3.12	1.38	0.55	0.79	1.25	1.17	0.98
DE - NL	-4.34	-4.47	-5.06	-2.96	-2.80	-3.42	0.25	0.46	0.25	0.75	1.64	0.58
DE - PT	-3.23	-2.94	-2.81	-6.12	-4.60	-4.20	1.01	0.99	0.44	0.85	1.95	1.12
DE - ES	-6.60	-4.30	-6.14	-5.95	-8.81	-7.16	0.96	1.98	0.70	1.18	1.33	0.93
DE - UK	-3.32	-3.27	-4.15	-4.04	-4.91	-7.80	1.17	1.64	1.77	0.63	1.63	0.16
DE - US	-4.68	-3.35	-3.13	-6.51	-6.34	-7.35	0.44	0.61	0.26	0.50	0.38	0.24
DE - IT	-8.38	-5.40	-7.06	-9.03	-5.76	-15.03	1.51	0.83	2.34	0.99	2.24	0.19
IE - NL	-2.17	-1.56	-2.11	-2.09	-1.79	-4.56	1.01	1.58	0.98	0.81	1.23	0.15
IE - PT	-3.17	-2.31	-2.77	-7.17	-3.55	-2.50	1.06	2.01	0.97	0.67	2.05	1.40
IE - ES	-2.20	-2.58	-3.19	-1.95	-2.12	-5.25	0.81	1.29	1.62	0.99	1.71	1.23
IE - UK	-1.66	-1.84	-2.27	-1.22	-1.78	-4.11	0.88	1.37	2.28	1.43	1.72	1.13
IE - US	-2.32	-2.06	-1.98	-2.51	-1.85	-2.63	1.32	1.20	1.80	0.38	1.77	1.64
IE - IT	-3.49	-2.73	-2.83	-3.52	-3.12	-4.60	0.43	0.99	1.22	1.88	0.92	0.43
NL - PT	-2.57	-2.40	-3.37	-4.86	-3.08	-4.47	1.05	1.30	0.69	0.42	1.42	0.91
NL - ES	-2.66	-4.11	-3.53	-2.66	-11.39	-3.83	0.58	3.78	0.67	0.64	1.87	0.27
NL - UK	-2.31	-2.56	-3.68	-3.38	-5.02	-7.18	1.60	2.61	3.92	0.07	1.17	1.25
NL - US	-3.28	-1.80	-2.10	-4.38	-3.97	-4.72	1.05	1.26	0.57	0.49	0.24	0.13
NL - IT	-5.59	-5.38	-5.62	-11.14	-9.14	-6.41	1.42	1.46	0.36	1.45	1.94	0.52
PT - ES	-3.54	-2.68	-3.75	-5.42	-4.67	-5.04	0.35	1.13	-0.27	1.23	0.83	0.38
PT - UK	-2.67	-3.56	-3.40	-8.10	-3.89	-8.11	2.89	0.58	0.48	0.96	1.98	0.52
PT - US	-2.69	-2.77	-3.15	-2.84	-4.22	-3.76	0.59	0.95	0.48	0.58	0.52	0.30
PT - IT	-3.57	-3.05	-3.46	-7.46	-4.02	-4.46	0.11	0.66	0.53	0.70	1.02	0.34
ES - UK	-3.18	-3.13	-3.61	-4.32	-5.14	-4.64	0.89	1.04	0.53	0.31	0.38	0.92
ES - US	-4.92	-3.49	-2.84	-3.26	-2.87	-3.36	-0.08	-0.50	-1.32	0.48	0.13	0.13
ES - IT	-4.91	-4.76	-4.87	-8.36	-4.79	-9.57	-0.90	0.66	0.42	0.84	1.93	0.71
UK - US	-2.59	-1.62	-1.51	-2.77	-2.93	-1.78	0.47	0.54	0.55	0.16	0.31	0.11
UK - IT	-4.77	-4.69	-4.21	-7.34	-10.74	-8.84	1.25	1.31	1.52	1.92	0.92	0.33
US - IT	-3.51	-3.02	-3.74	-7.70	-6.03	-3.48	0.49	1.03	0.50	0.08	0.17	0.05

Table 3 (continued): The t-tests for equality in extreme dependence for the stock market returns in US dollars. The first and second columns report the result of the t-values testing whether lower and upper tail dependence change significantly after the introduction of the euro. Values below -1.96 (above 1.96) indicate that extreme dependence becomes significantly larger (smaller) after the formation of the EMU at the 5% level of significance. The third and fourth columns report the t-values testing the null that upper and lower tail dependence are equal in the pre-EMU or the EMU period. Values below -1.96 indicate that upper tail dependence is significantly more pronounced than lower tail dependence and values above 1.96 indicate that lower tail dependence is significantly more pronounced than upper tail dependence in the respective period.

Portfolio size	Pre-EMU period		EMU period	
	Lower tail	Upper tail	Lower tail	Upper tail
2	0.78	0.97	0.54	0.75
3	0.36	0.41	0.21	0.28
4	0.36	0.45	0.18	0.28
5	0.37	0.49	0.19	0.28
6	0.40	0.51	0.18	0.29
7	0.41	0.54	0.16	0.28
8	0.43	0.55	0.17	0.29
9	0.46	0.59	0.19	0.30

Table 4: Share of portfolios with asymmetrical extreme dependence for different portfolio sizes.